Financial Engineering

Modeling Methodology & Applications

財務工程之模型建構與應用

Jenher Jeng, Adjunct Professor at National Chiao-Tung University (NCTU)
鄭振和 國立交通大學兼任教授 (應數系/財金所/國家理論中心數學組)
2004 ~ 2010

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Signal Processing

Data Analysis

Transformation, Filters, Regressors, Classification, Co-integrators

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Signal Processing

“The name of the game then was distinguishing the signal from the noise, which was often difficult. The key question on my mind was typically: What part of each monthly observation on inflation is durable and what part is fleeting?”

- Former Vice Chairman of Federal Reserve Bank Alan Blinder

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Signal Processing

Example - I

Large Time-Scale S&P 500

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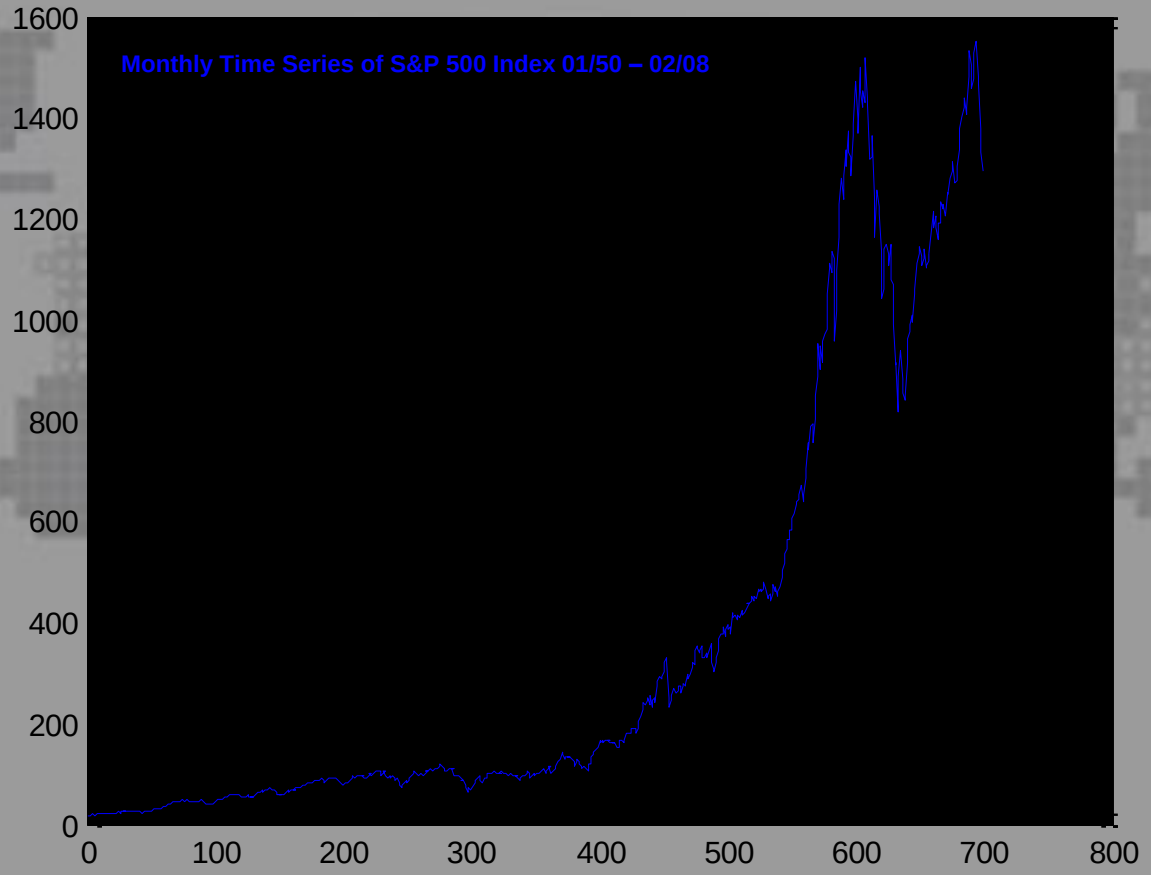

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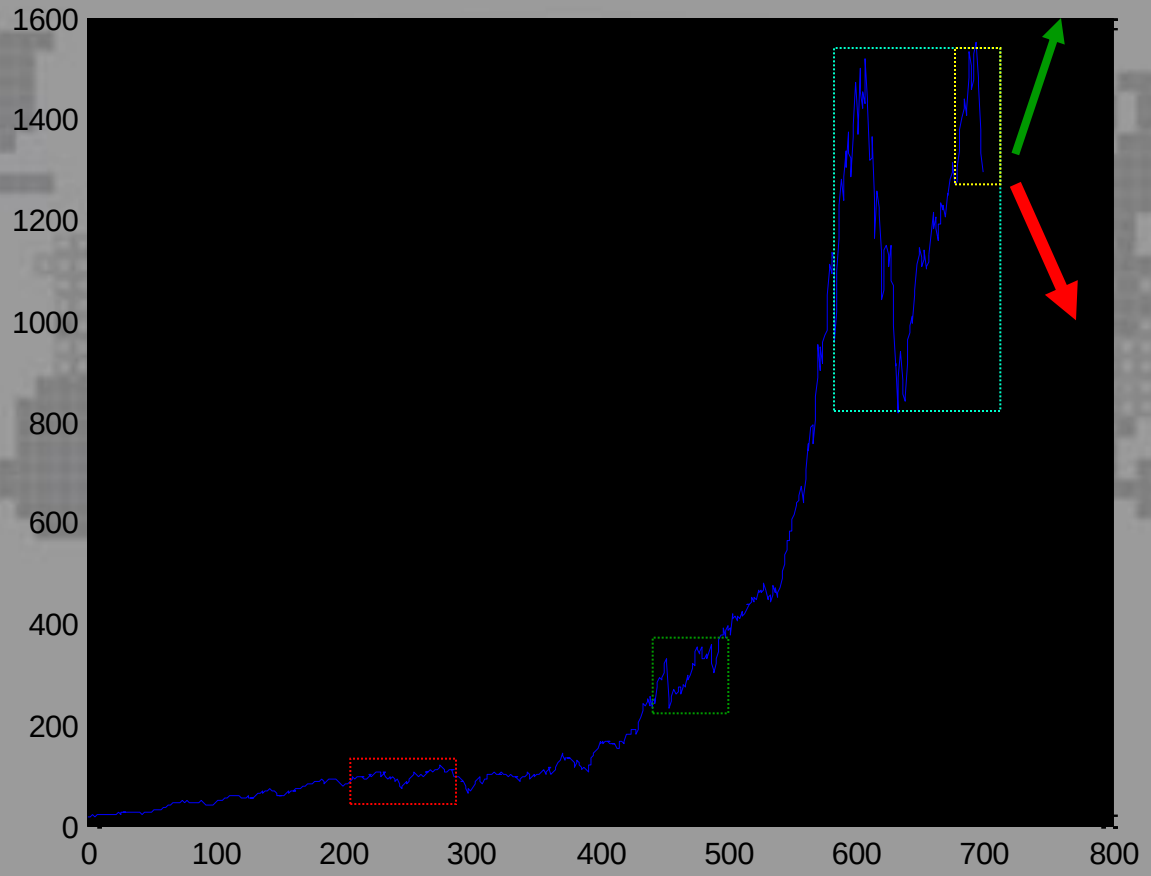

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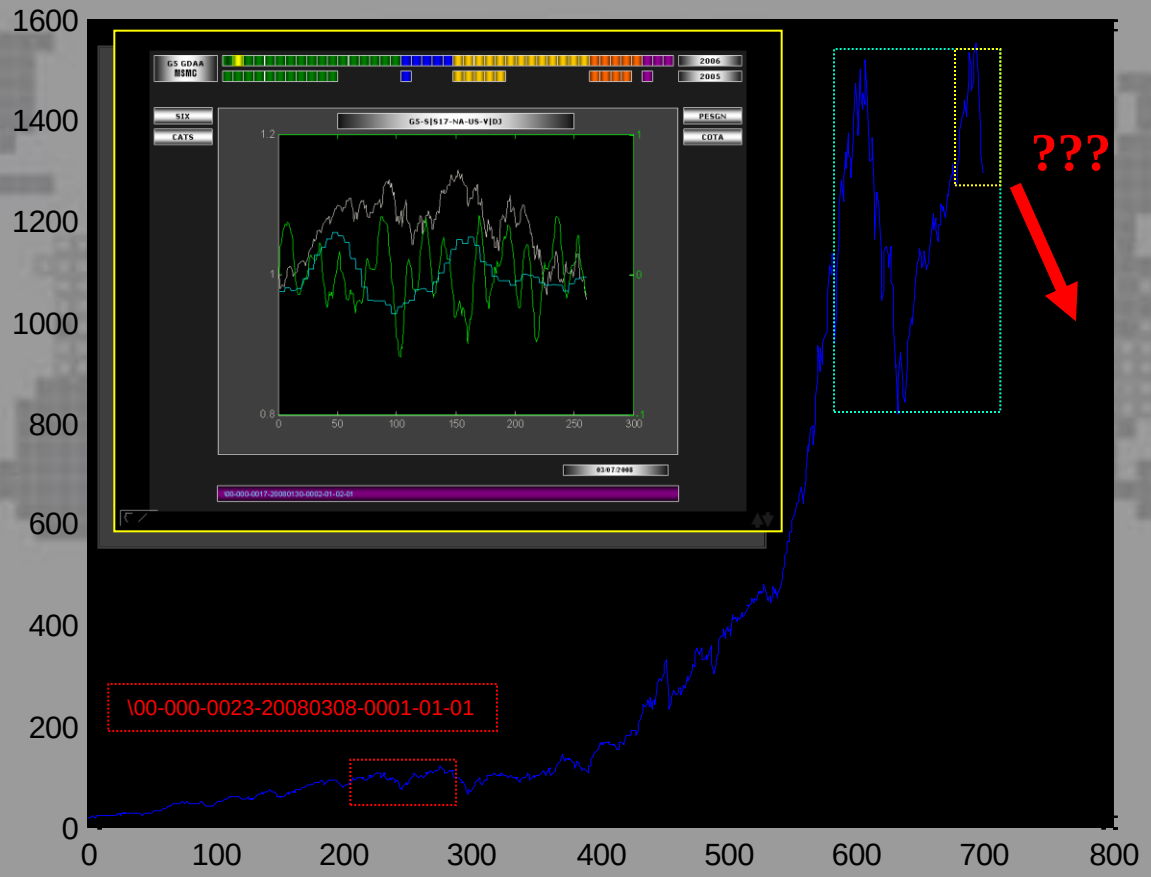
Financial Signal Processing for justifying Experiences



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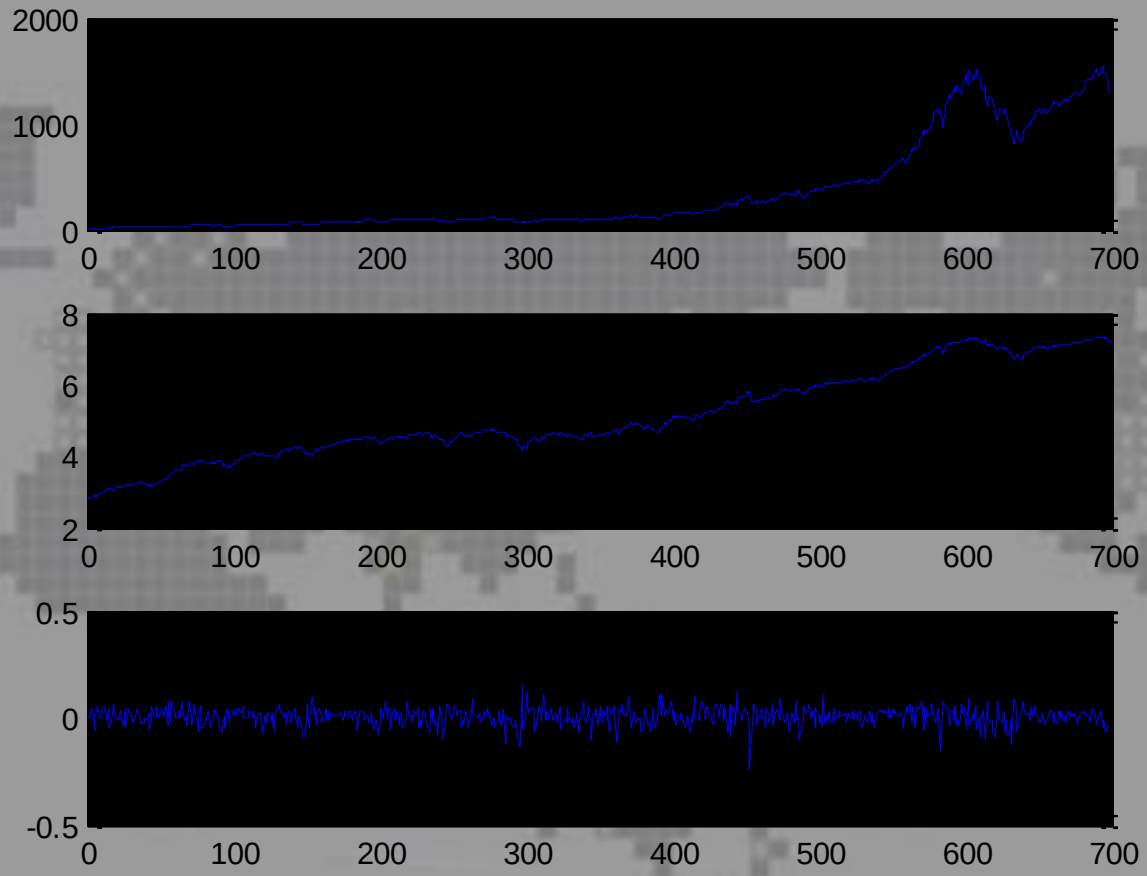
Difference of Signal Processing from Technical Analysis

just like difference of Astronomy from Astrology



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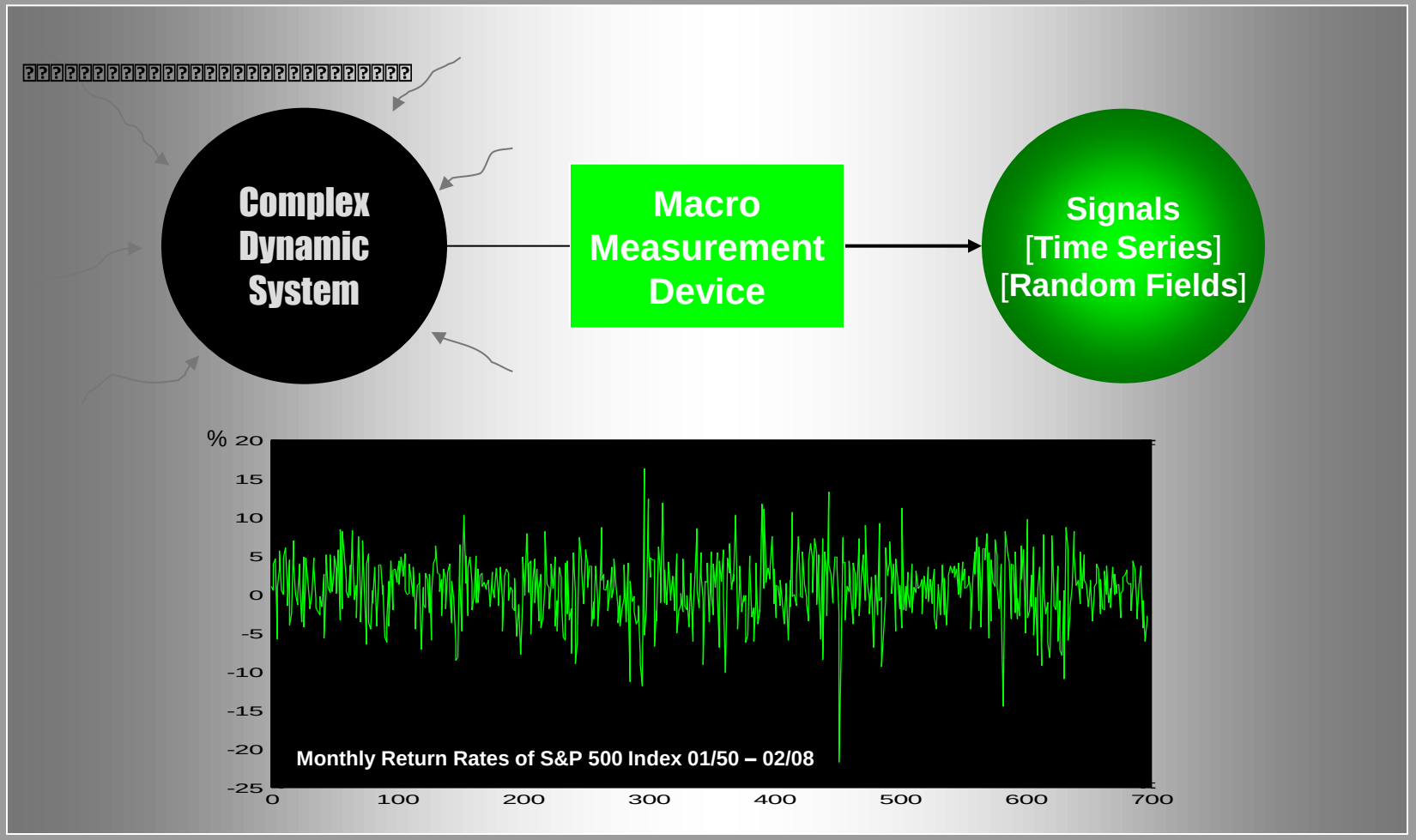
Tracking Capital Market as Complex Dynamic System



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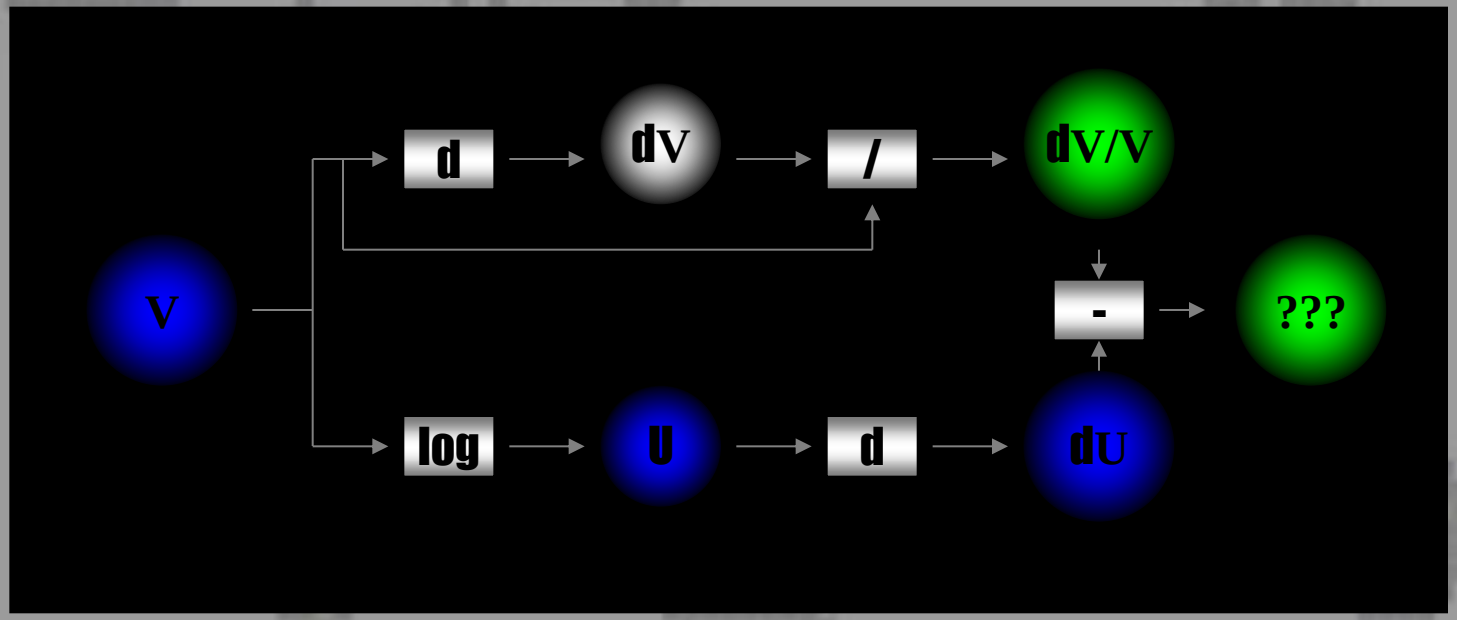
Constructive Math (Operator-Format) for Signal Processing



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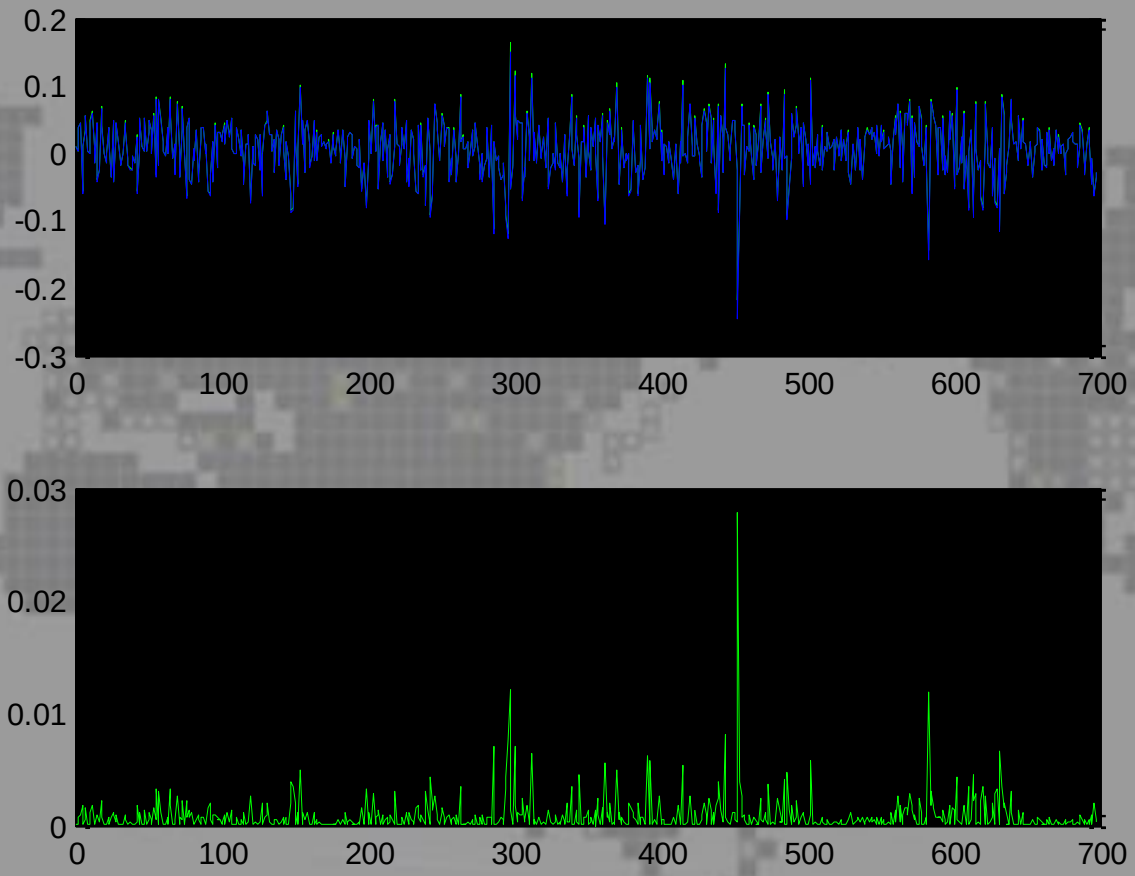
Math Magic for revealing latent Volatility



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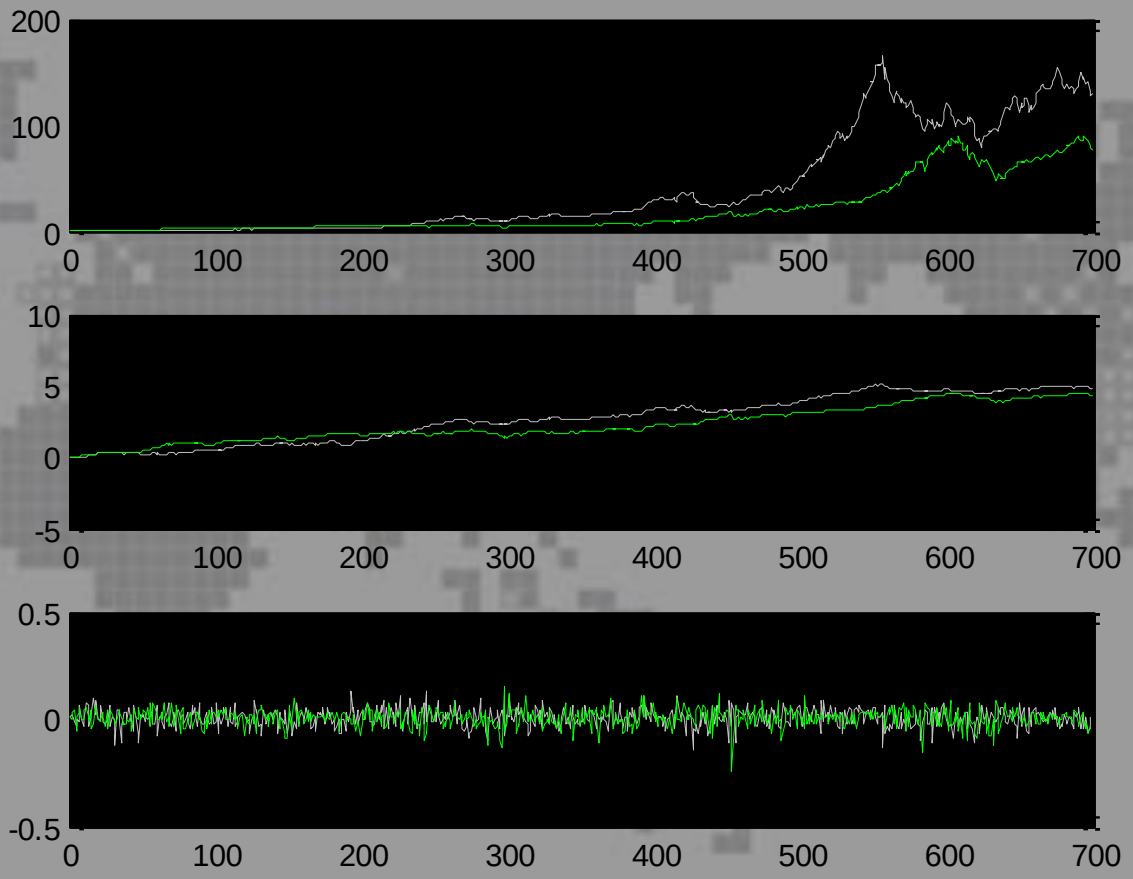

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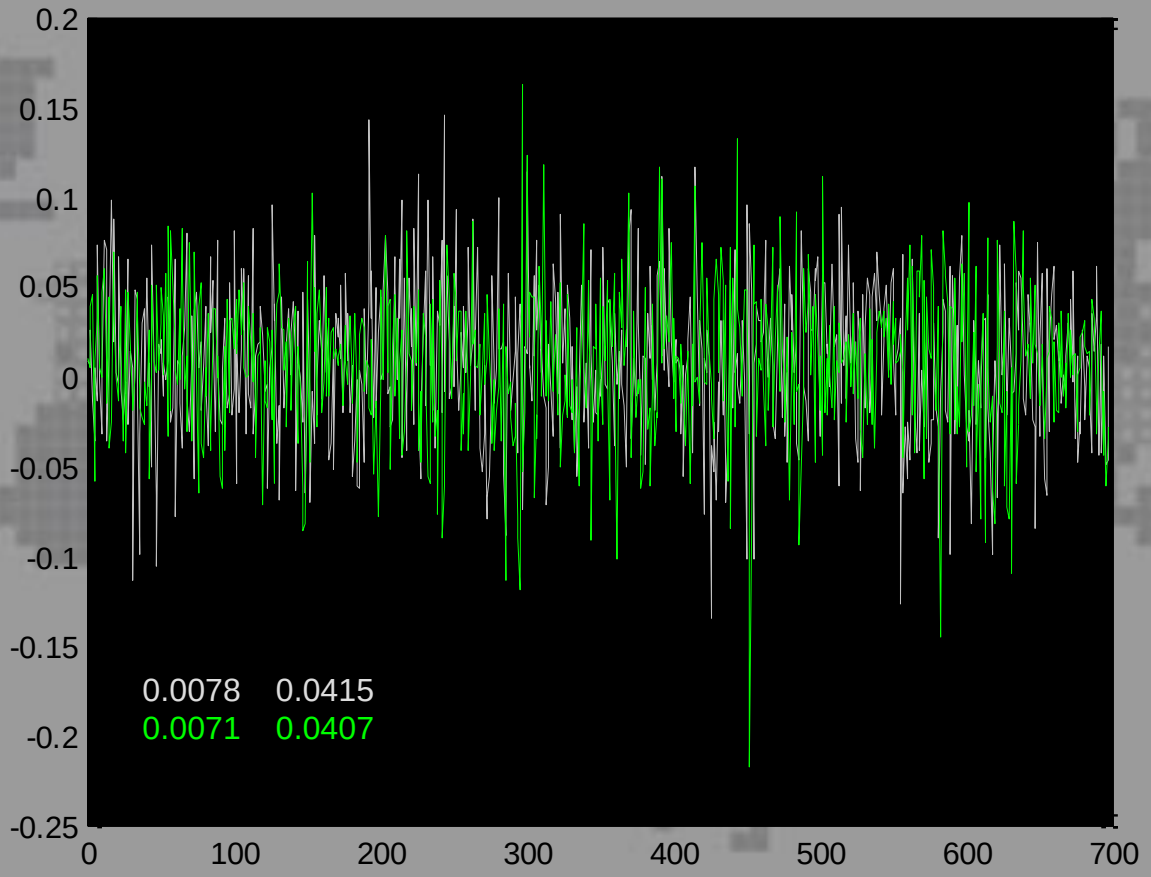

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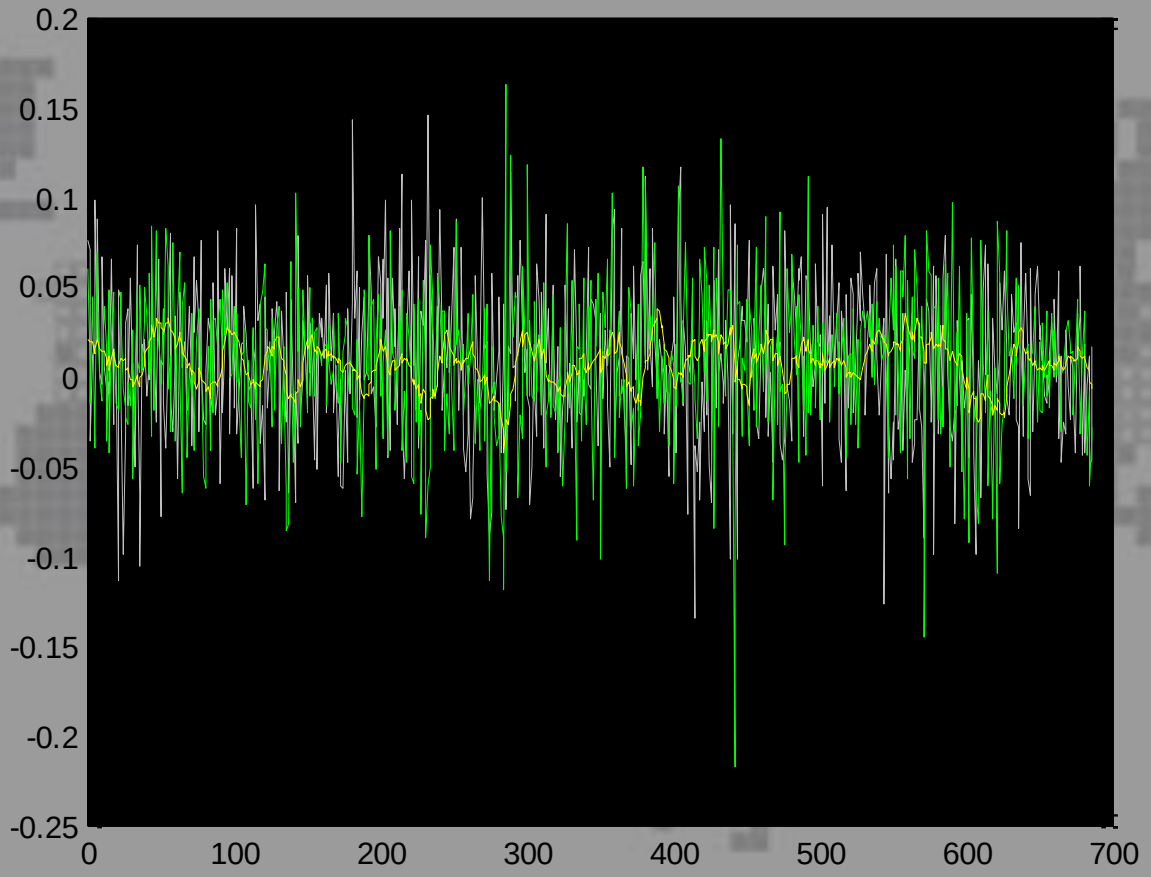

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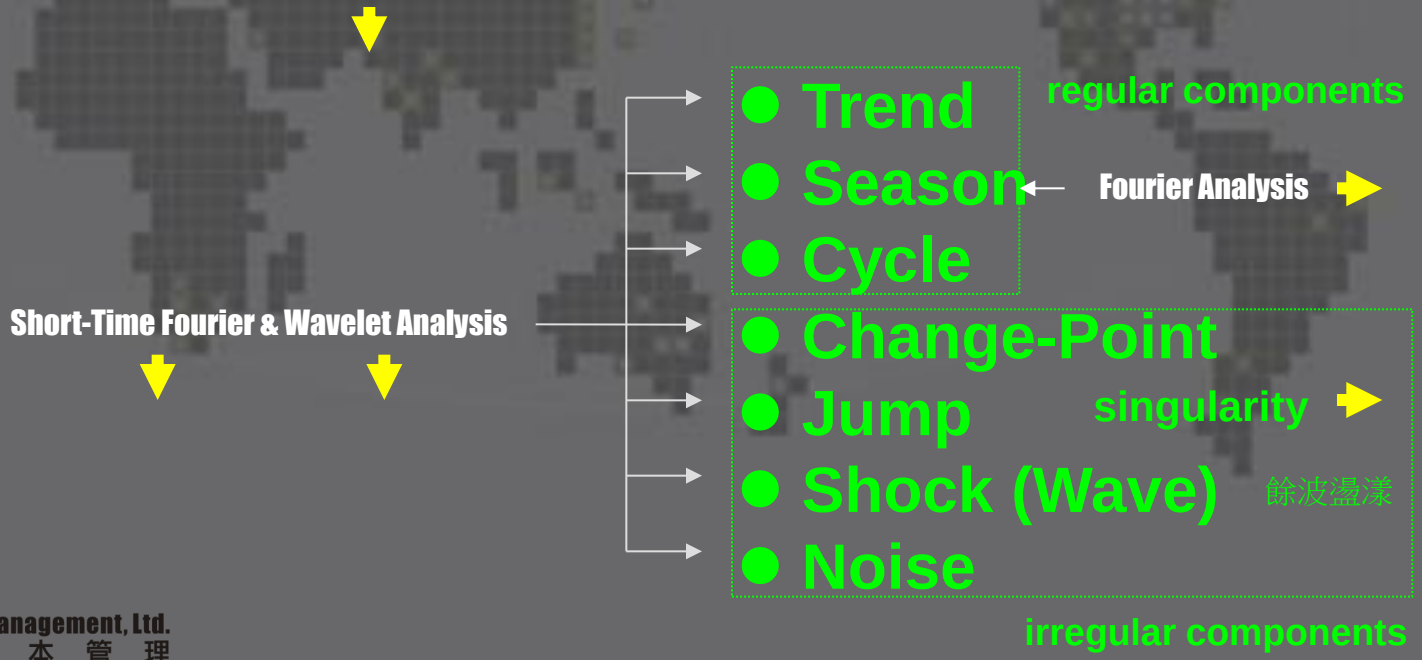
Signal Processing Task before modeling Complex Dynamic System

By Reflexivity,

no single simple rule will dominate the stock market index forever.

Signal processing decomposing the constitutional components and exploring the structural nature of the signal.



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Signal Processing

Example - II

Crude Oil in Ultra-High Frequency

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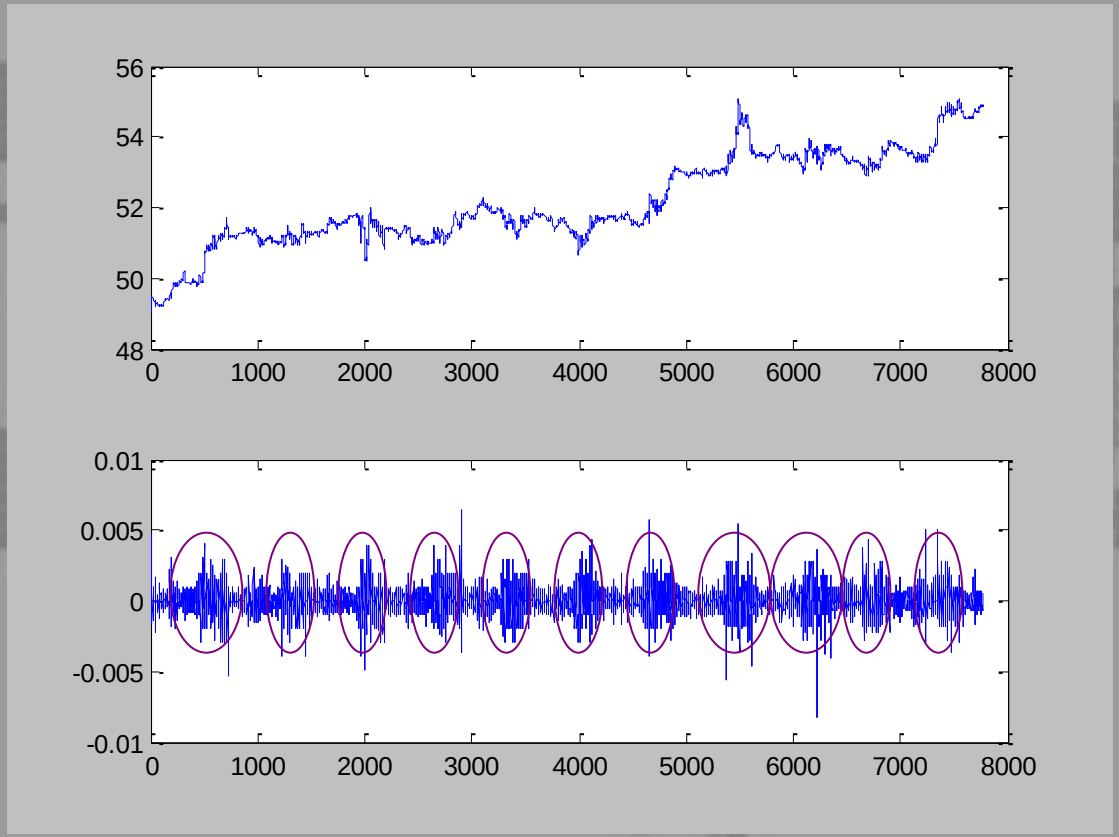

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Signal Processing

Example - III

TAIEX Future-Spot Return-Rate Spread

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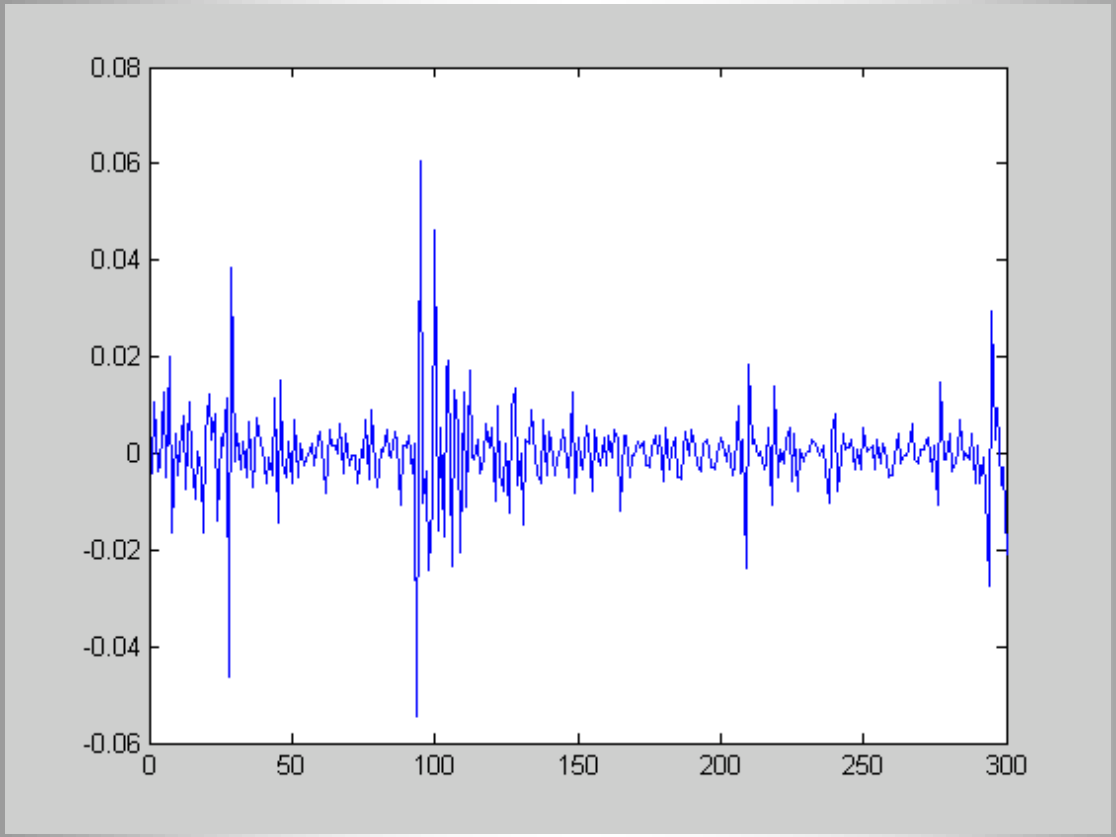
Future-Spot Return-Rate Spread



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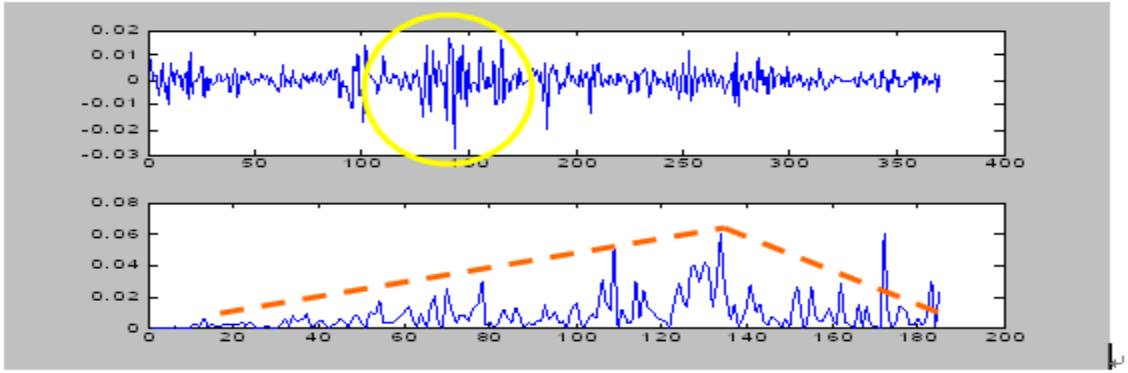
Analog in Structural Nature of Signal



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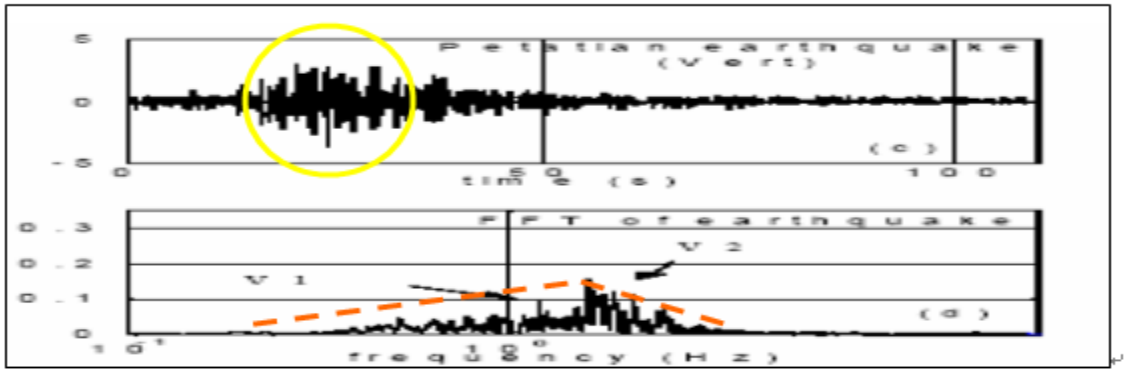

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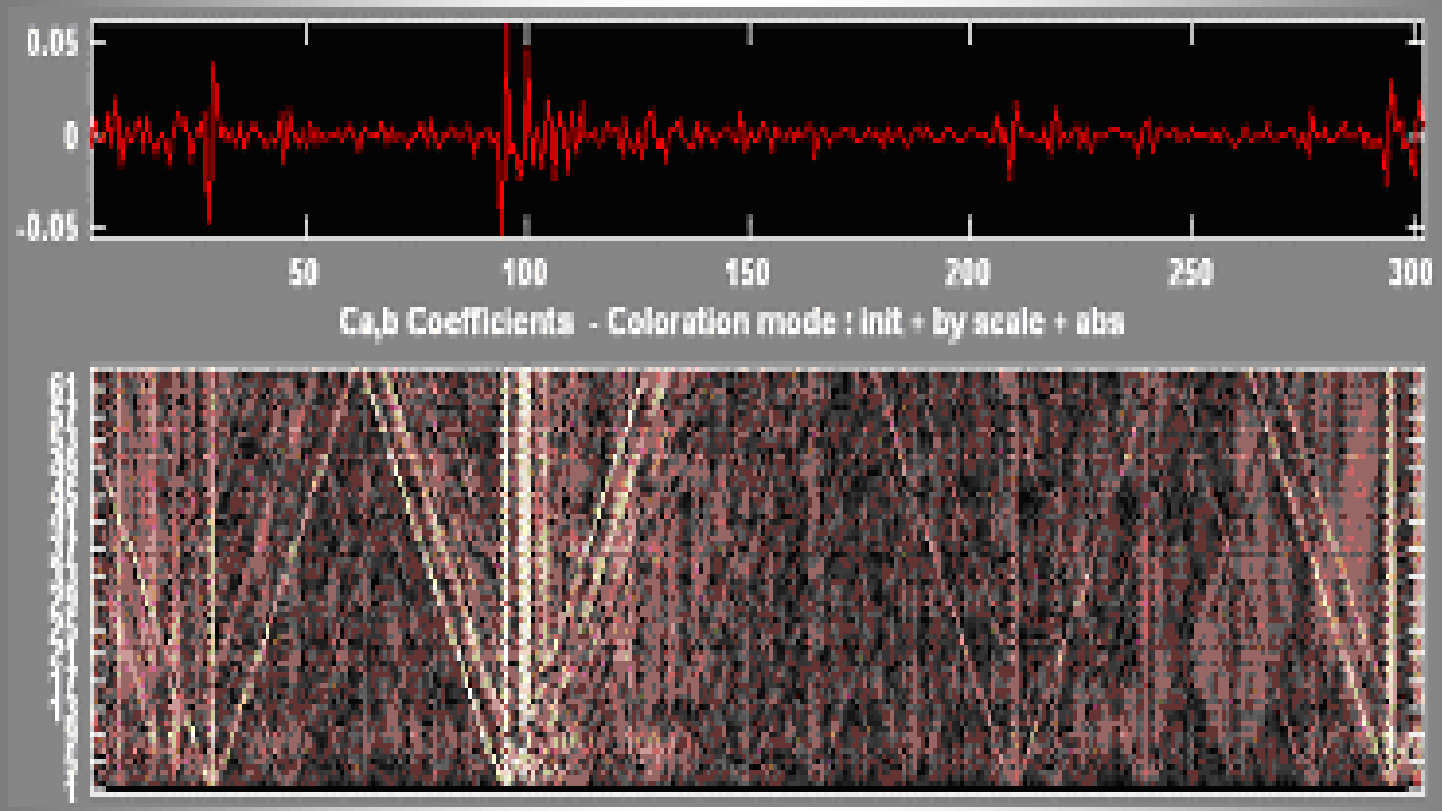
Continuous Wavelet Transform



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Signal Processing

Example - IV

Global Temperature

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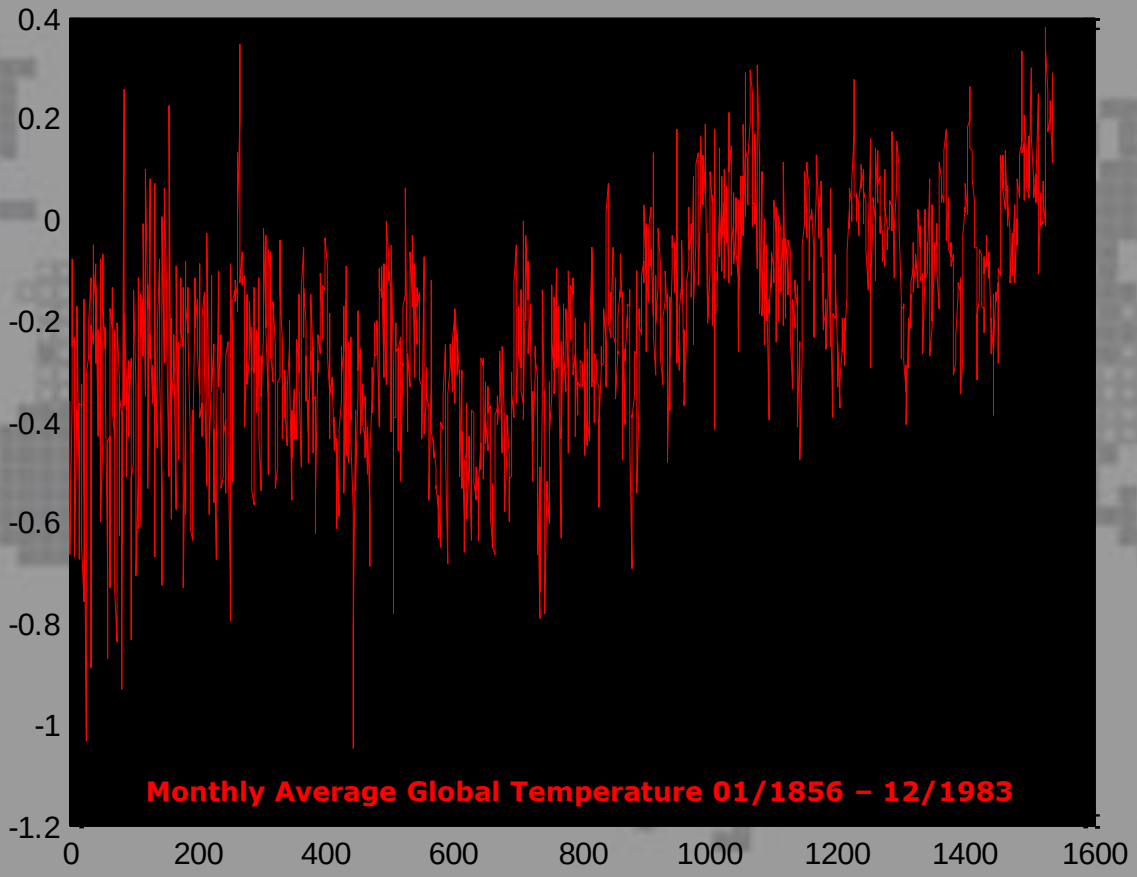
Global Warming



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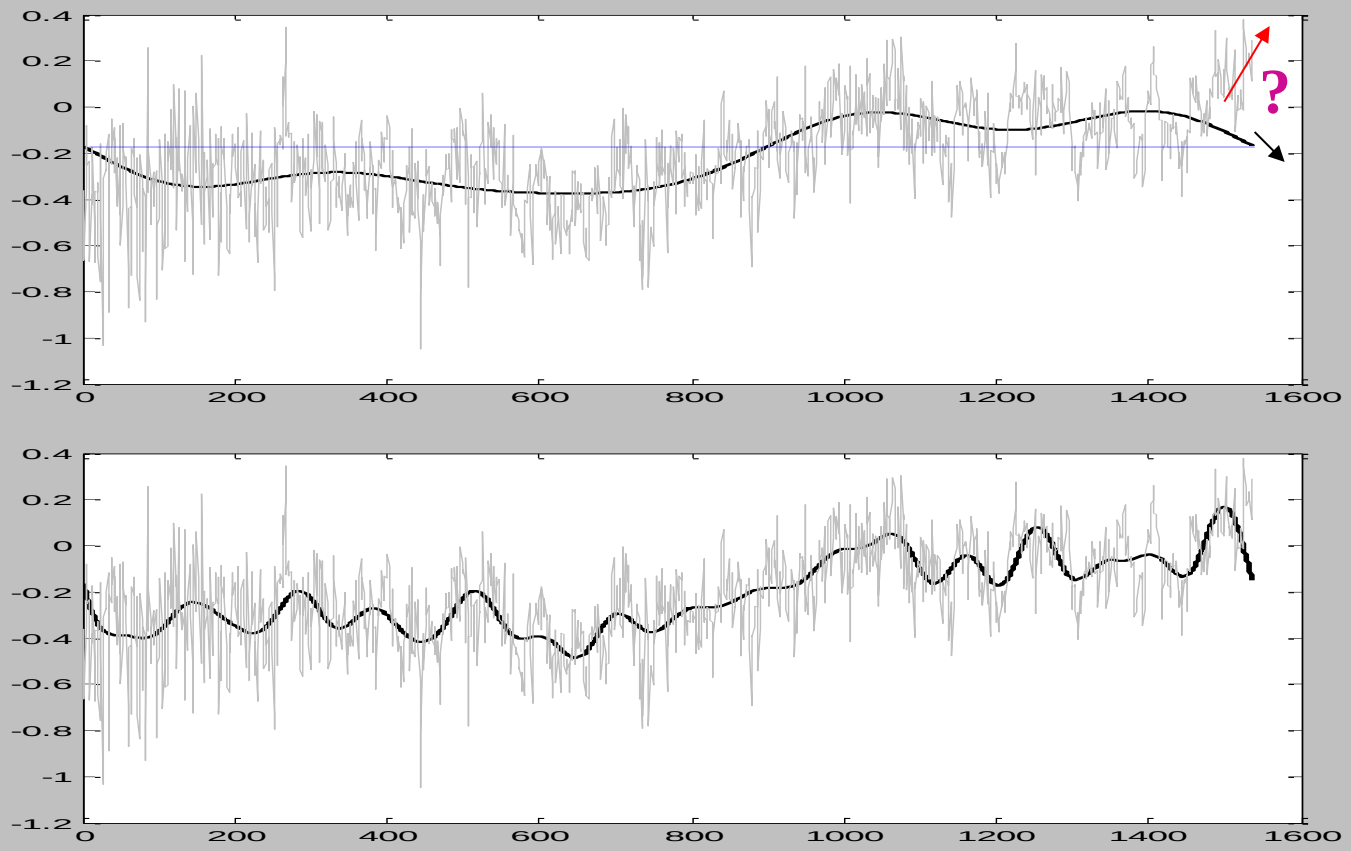
Trend of Global Warming



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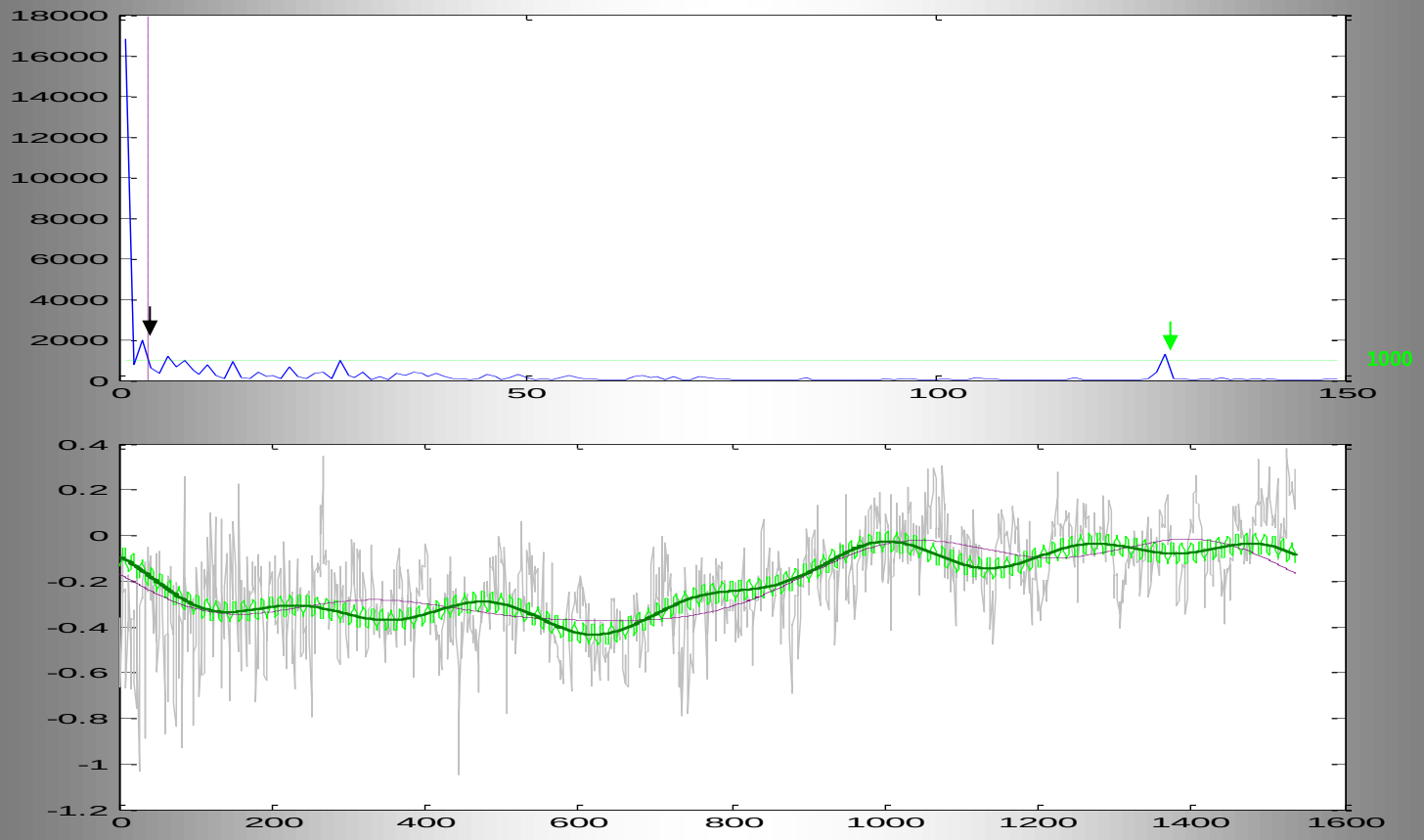
Trend of Global Warming



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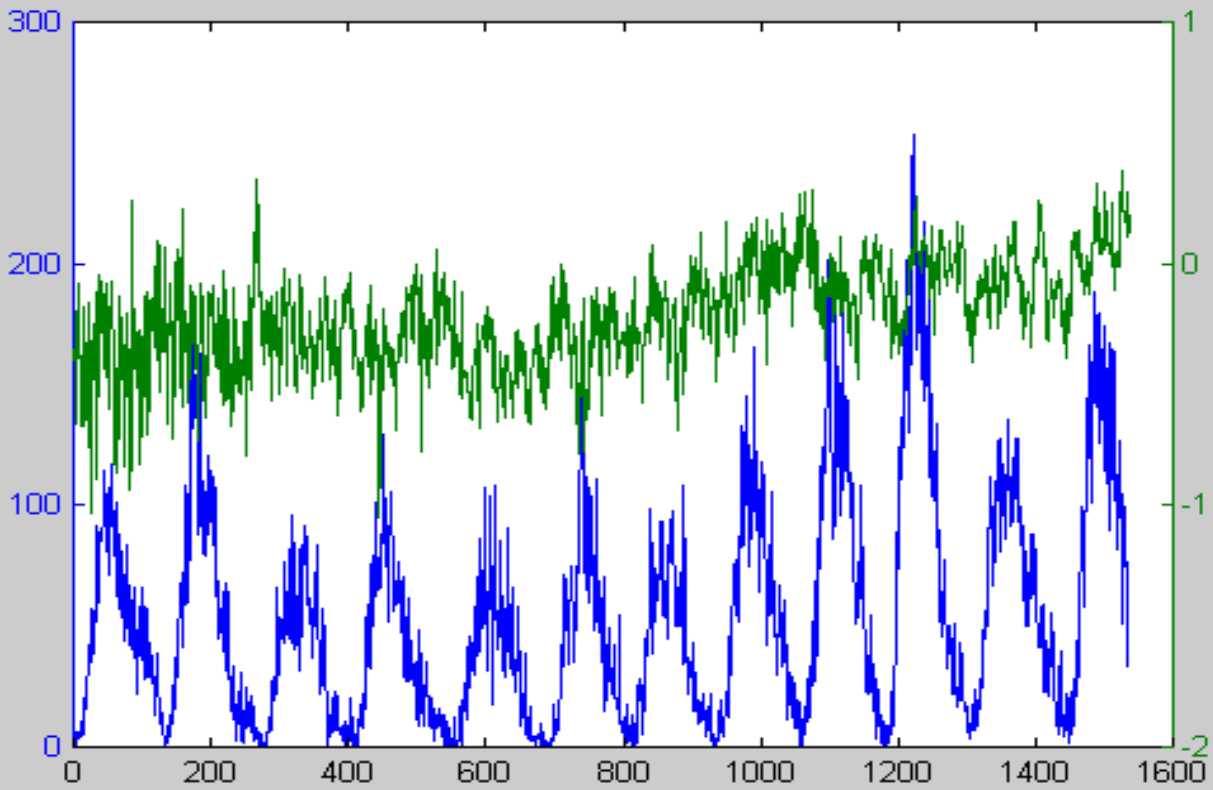
Trend of Global Warming



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De-noising Puzzle of Signal Processing

Observed Time Series = Signals + Noises

in

GWN Modeling

If we a priori know the structure of the underlying signal,

we would know how to nicely get the de-noising job done.

But we usually do not have much information about that

until we are somehow doing it properly.

Foundation of Signal Processing – Fourier Analysis

$$\hat{f}(\omega) = \int_R f(t) e^{-i\omega t} dt, \quad \omega \in R.$$

Duel Identities of the Signal

$$f(t) = \frac{1}{2\pi} \int_R \hat{f}(\omega) e^{it\omega} d\omega, \quad t \in R.$$

Foundation of Signal processing – Fourier Analysis

Fourier Transform $\hat{f}(\omega) = \int_R f(t) e^{-i\omega t} dt, \quad \omega \in R.$

Duel Identities of the Signal

?

$$f(t) = \frac{1}{2\pi} \int_R \hat{f}(\omega) e^{it\omega} d\omega, \quad t \in R.$$

Redundancy Reduction

f 2π-periodic

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-\mathrm{int}} dt.$$

$\omega \in N$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\mathrm{int}}$$

Fourier Series

Discrete Fourier Transform

$$\gamma_k = \frac{1}{N} \sum_{j=0}^{N-1} f\left(\frac{2\pi j}{N}\right) e^{-ik\frac{2\pi j}{N}}, \quad k = 0, \ldots, N-1.$$

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Singularity vs/& Irregularity

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Foundation of analyzing Signal Structure

Exploring Taylor's Expansion

$$f(t) = \sum_{i=0}^{k} \frac{f^{(i)}(t_0)}{i!}(t - t_0)^i + R_k(t_0, t), \quad t \in (t_0 - \varepsilon, t_0 + \varepsilon).$$

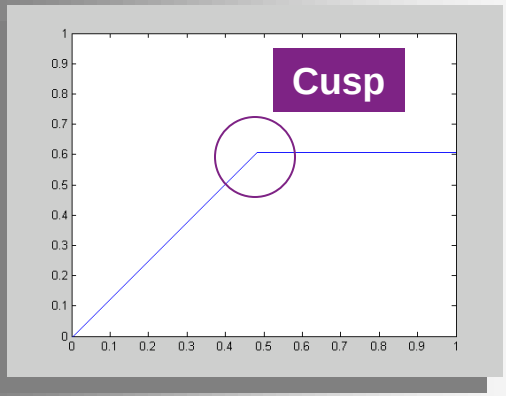
$R^{(i)}$

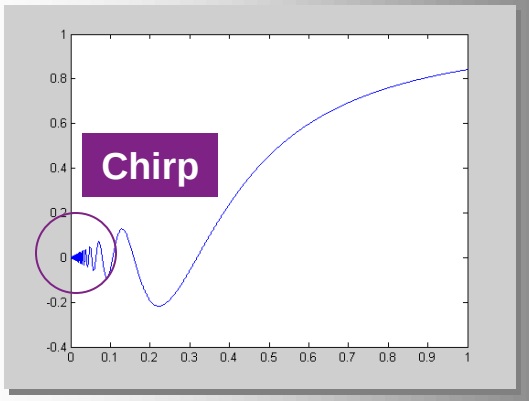
f *is smooth as* $R^{(i)} \downarrow 0$ *as* $i \uparrow \infty$.

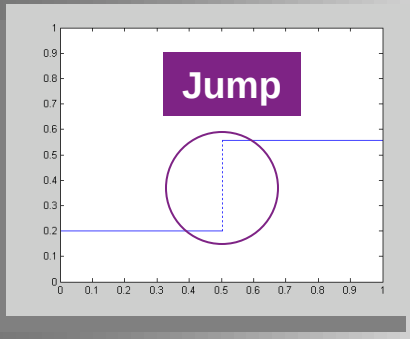
f *is wild as* $R^{(i)} \uparrow \infty$ *as* $i \uparrow \infty$.

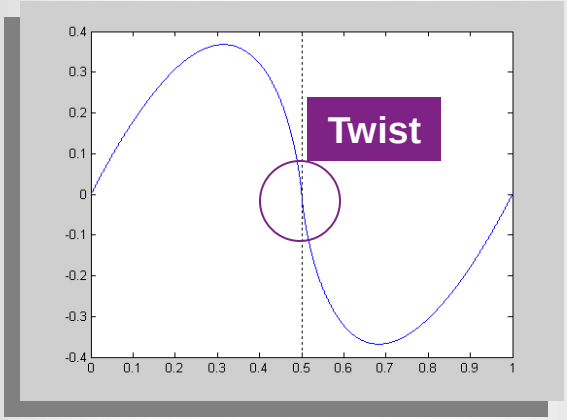
f *is singular as* $R^{(i)} = \infty$ *at some i.*

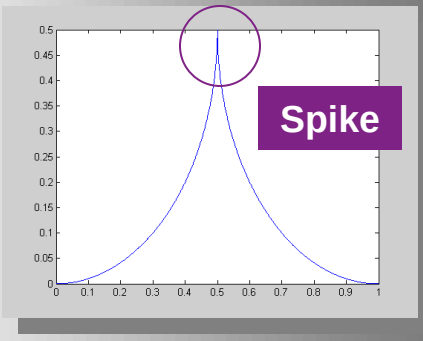
Types of Singularity











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Foundation of analyzing Noisy Behaviors

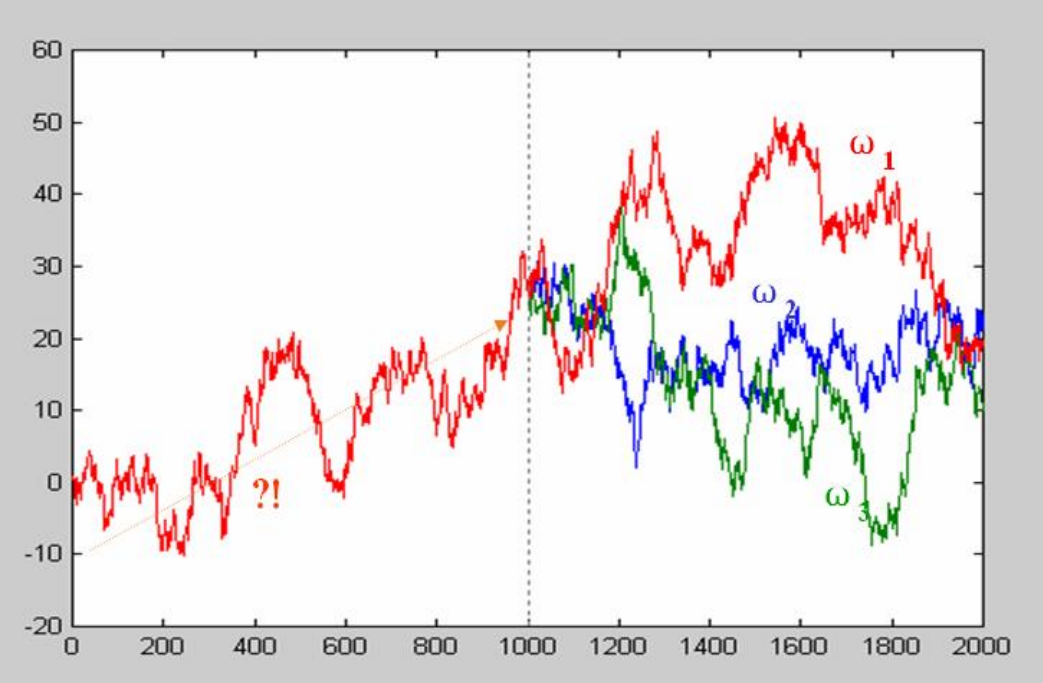
$$\Delta X_i^n \equiv X_i^n - X_{i-1}^n = [a^n(t_i) \cdot \chi_i^n] \cdot \nu_i^n(\Delta t)$$

where $\Delta t = 1/n$, a^n is real-valued, ν_i^n is monotone increasing, and χ_i^n is a real-valued random variable.

Remark. Note that $[a^n(t_i) \cdot \chi_i^n]$ can be regarded as the stochastic fractal changing rate.

Foundation of analyzing Noisy Behaviors

$$\Delta X_i^n \equiv X_i^n - X_{i-1}^n = [a^n(t_i) \cdot \chi_i^n] \cdot \nu_i^n(\Delta t)$$



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Examples of Uncertain Fluctuation
I

$$\Delta X_i^n \equiv X_i^n - X_{i-1}^n = [a^n(t_i) \cdot \chi_i^n] \cdot \nu_i^n(\Delta t)$$

Deterministic Smooth Motion

Consider the trivial case where $\chi_i^n = 1$ with probability 1, $\nu_i^n(x) = x$ and

$a^n = a$ for all i and n. Then F_t is a smooth path with the derivative $a(t)$.

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Examples of Uncertain Fluctuation II

$$\Delta X_i^n \equiv X_i^n - X_{i-1}^n = [a^n(t_i) \cdot \chi_i^n] \cdot v_i^n(\Delta t)$$

Lévy Class – (constant in a, homogeneous in v, and χ_i^n's i.i.d.)

Set $a = 1$ without loss of generality.

- ***Wiener Process (Brownian Motion)***

  Consider the simple nontrivial case where $v_i^n(x) = x^{1/2}$, and

$$\chi_i^n = \begin{cases} = +1 & \text{with probability } \frac{1}{2}; \\ = -1 & \text{with probability } \frac{1}{2} \end{cases}$$

  for all i and n.

Examples of Uncertain Fluctuation II

$$\Delta X_i^n \equiv X_i^n - X_{i-1}^n = [a^n(t_i) \cdot \chi_i^n] \cdot \nu_i^n(\Delta t)$$

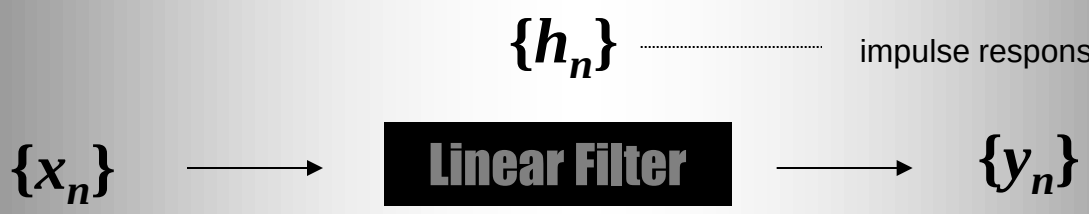
- *Poisson Process*

Consider the extreme case where $\nu_i^n(x) = x^0 \equiv 1$, and

$$\chi_i^n = \begin{cases} = 1 & \text{with probability } \lambda \cdot \Delta t, \\ = 0 & \text{with probability } 1 - \lambda \cdot \Delta t \end{cases}$$

for all i and n.

Basic Framework of filtering Signal



$$y_n = \sum_i h_{n-i} x_i$$

$$Y(\omega) = H(\omega) \cdot X(\omega)$$

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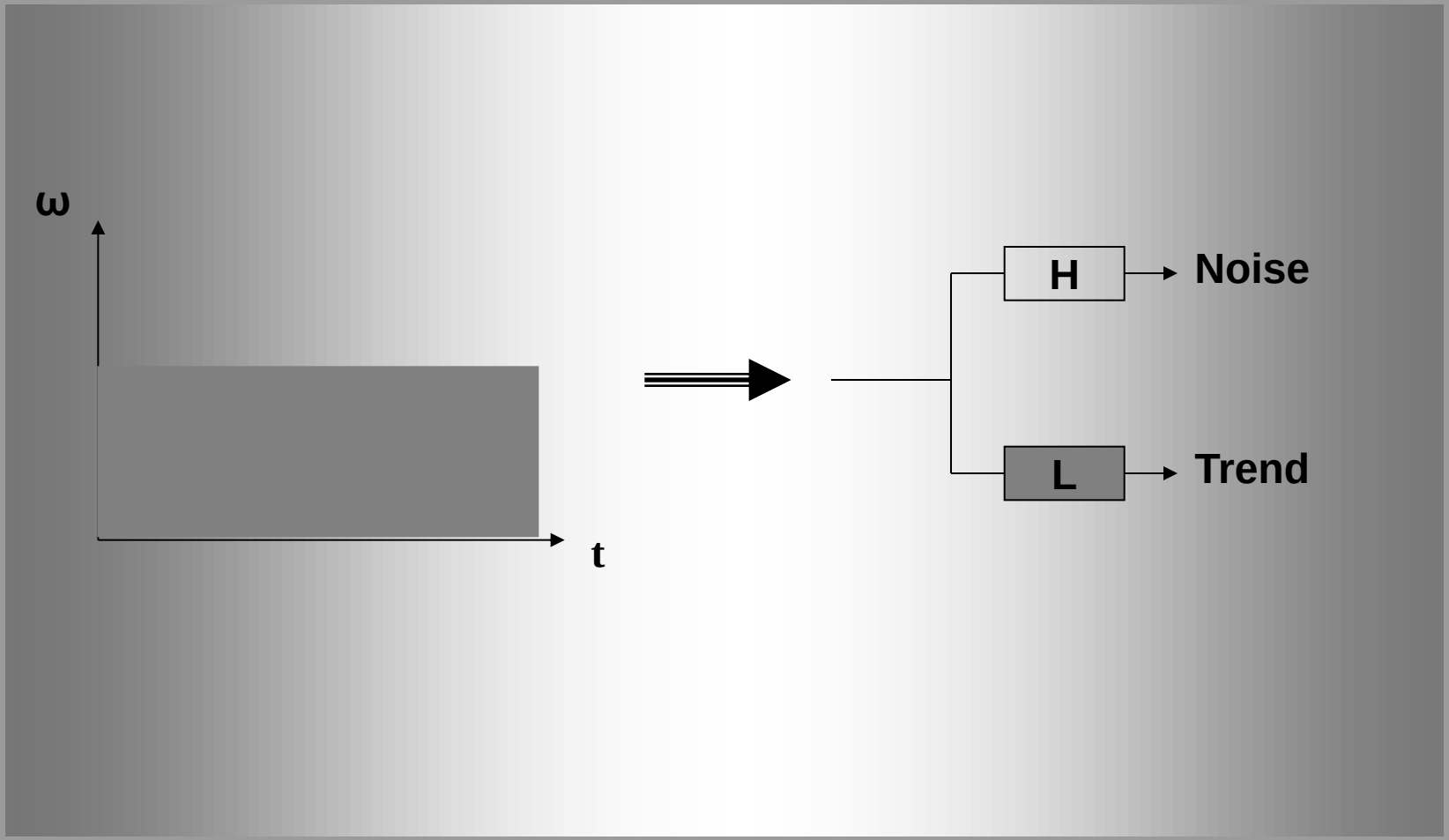
Basic Framework of filtering Signal



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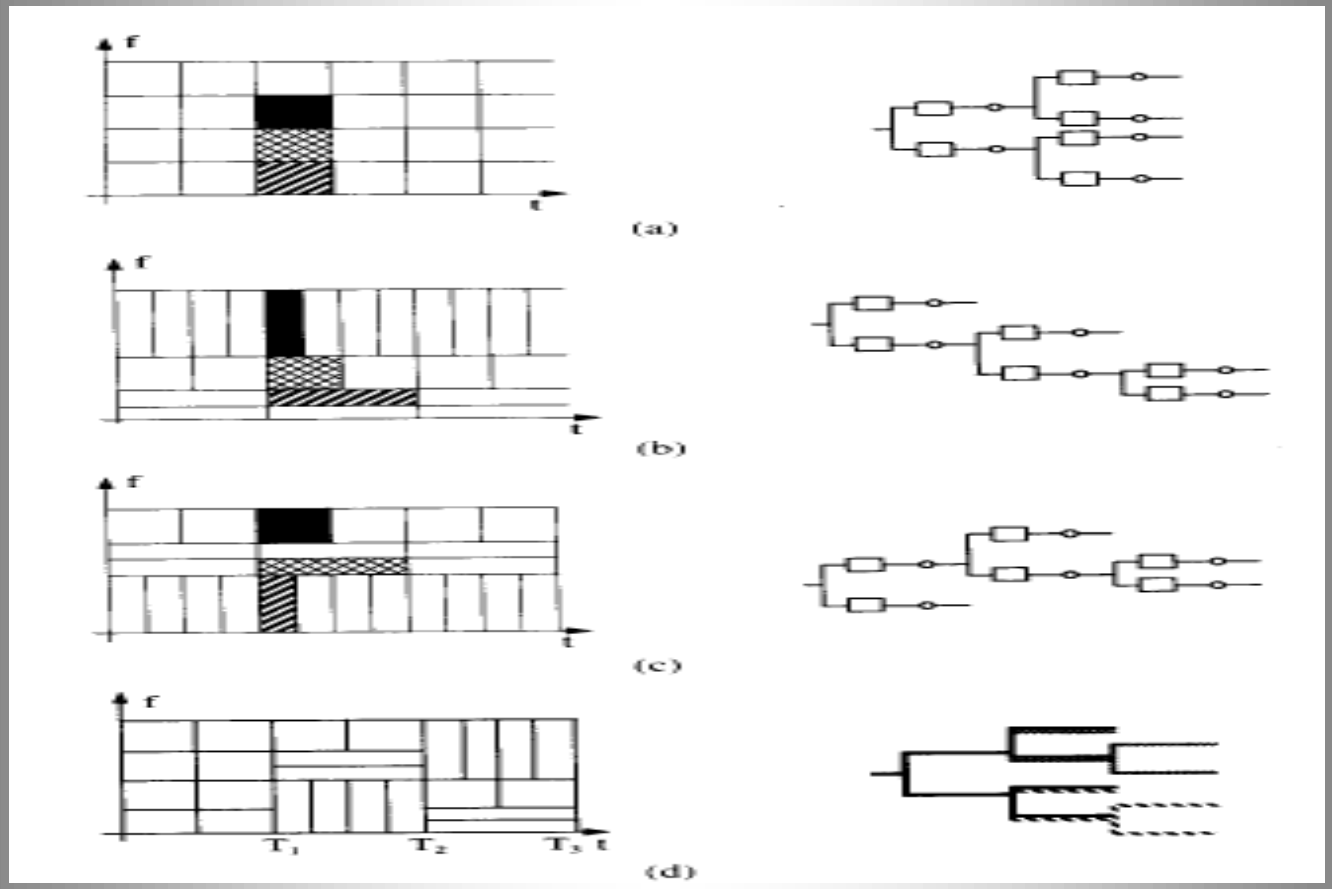
Sophisticated Framework of filtering Signal



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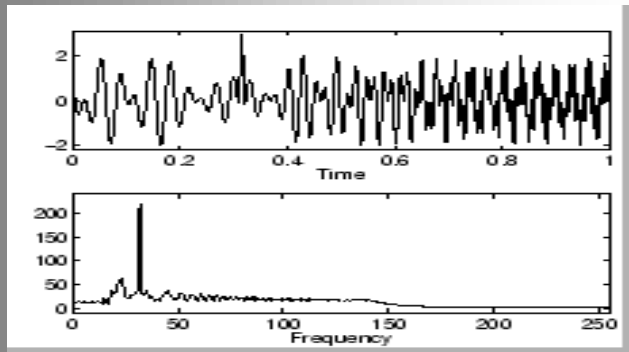
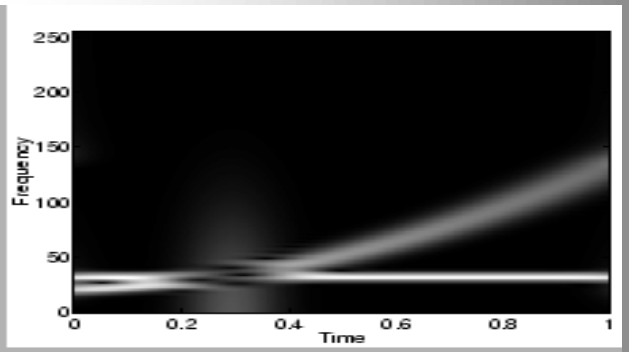
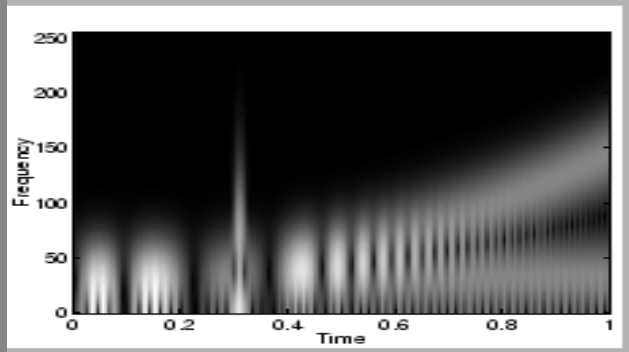
Short-Time Fourier Transform



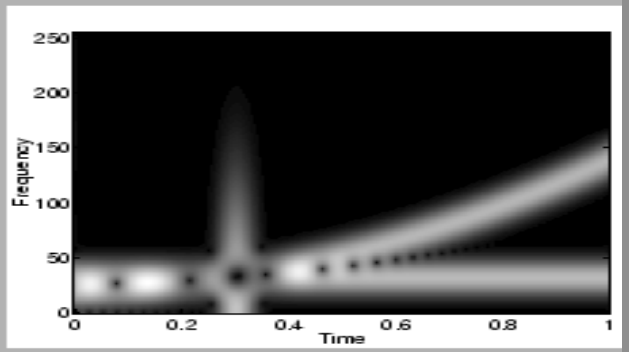
(a) Signal and its Fourier transform



(b) STFT with wide window



(c) STFT with narrow window



(d) STFT with medium window

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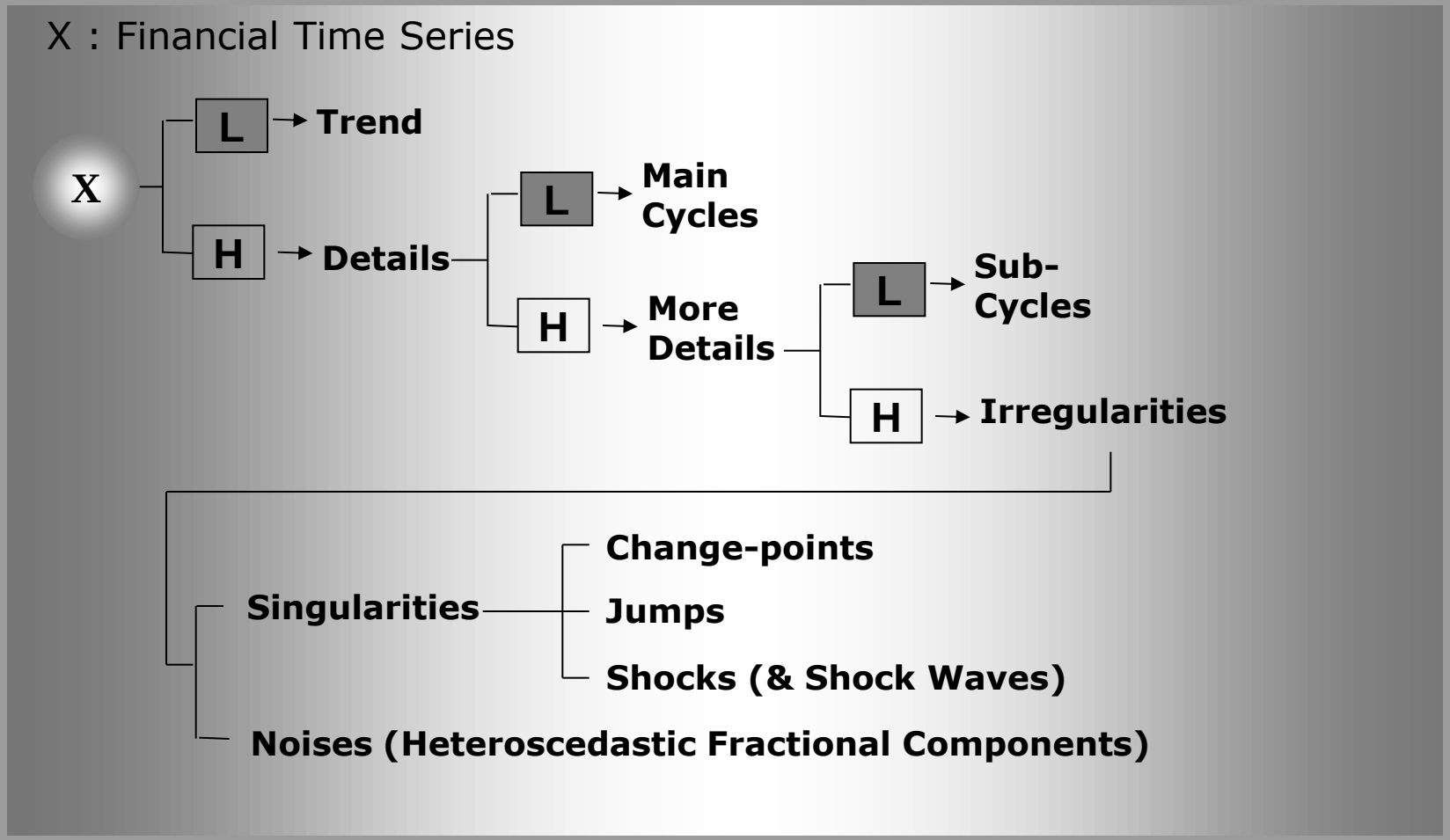
Multi-resolution Power of analyzing Financial Signal



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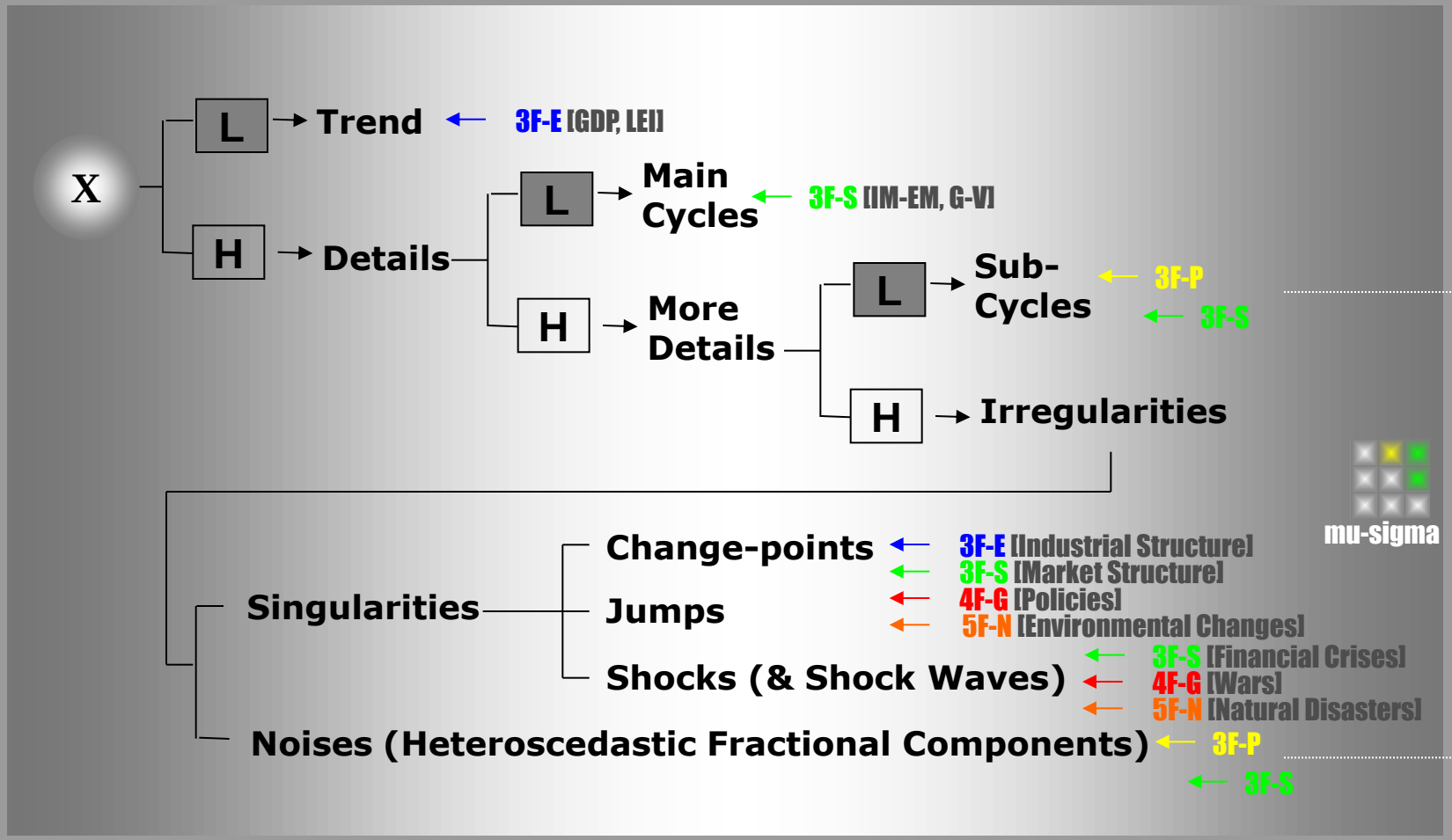
Dynamic Relations of Multi-resolutional Components

& Integrated* Risk Factor Analysis CMEK



various filter functions for emphasizing on capturing different patterns

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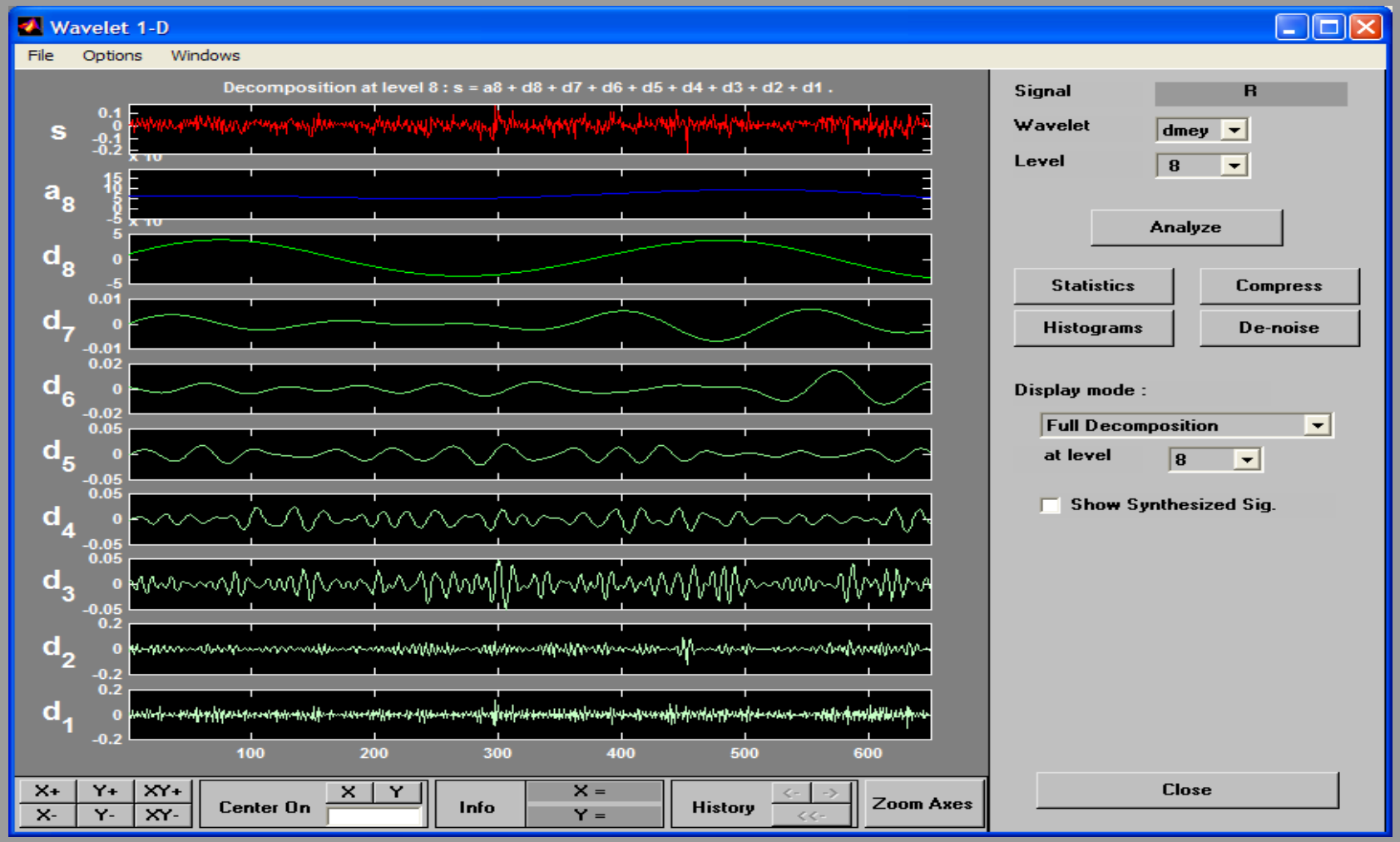

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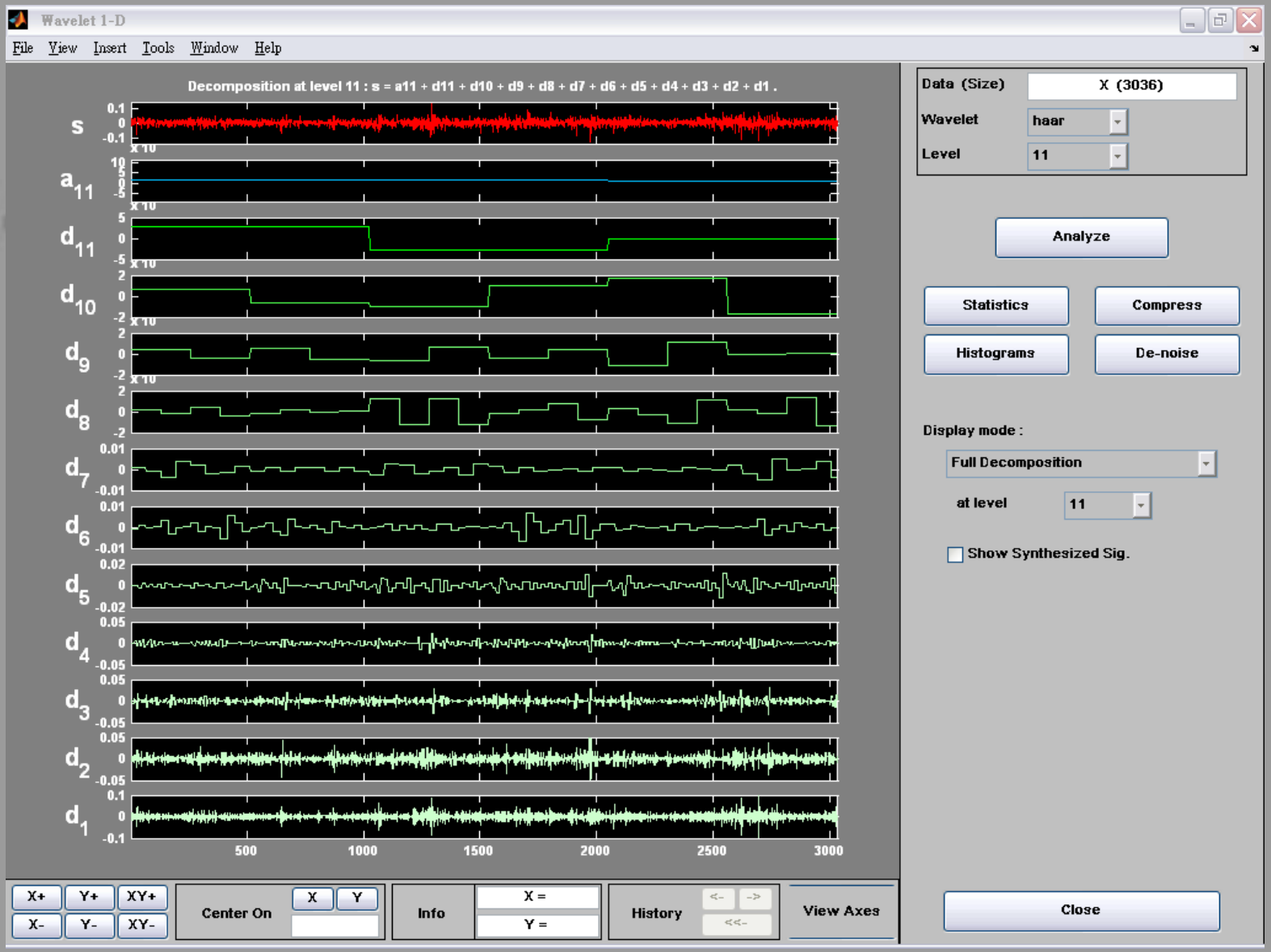

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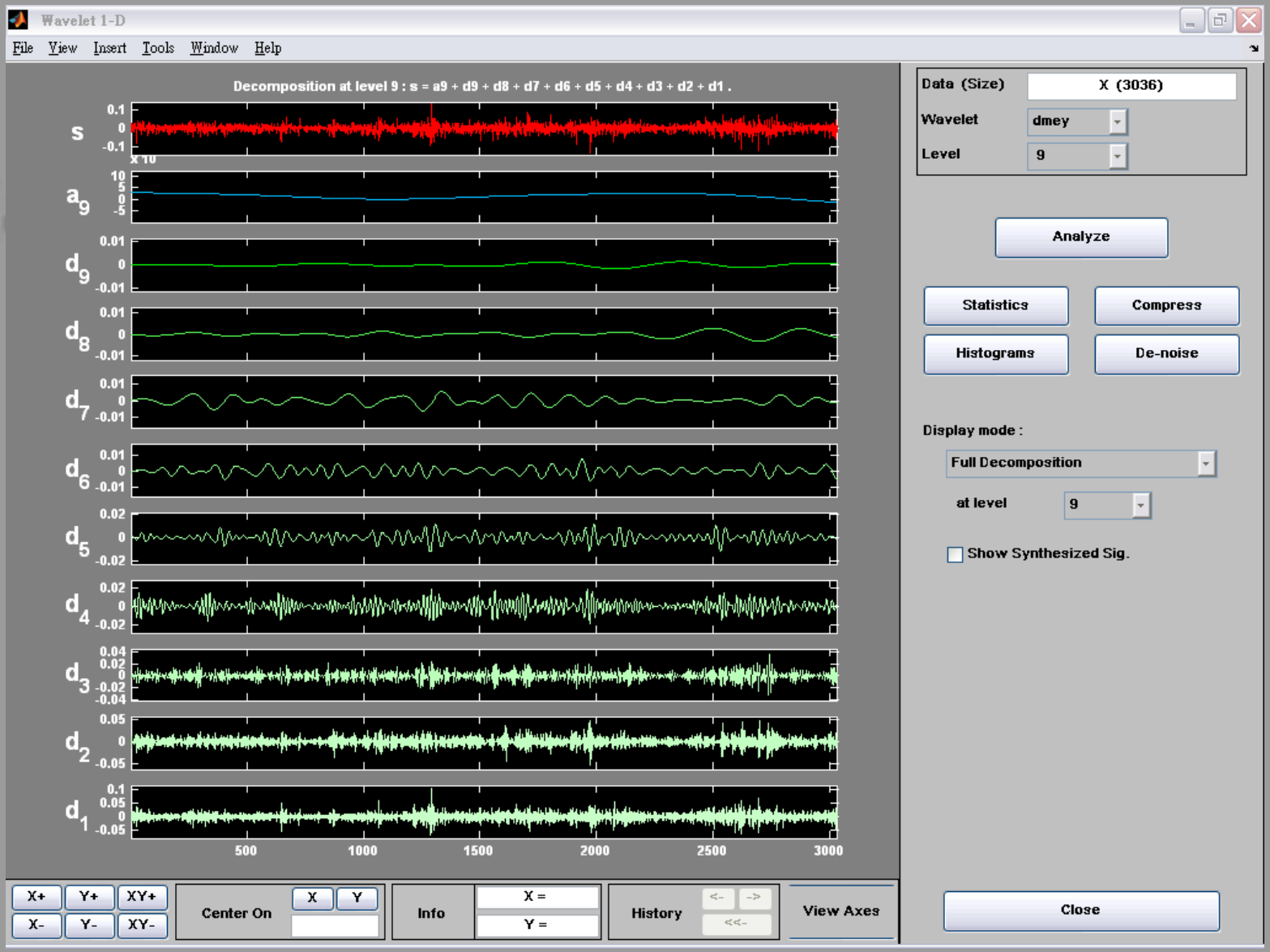

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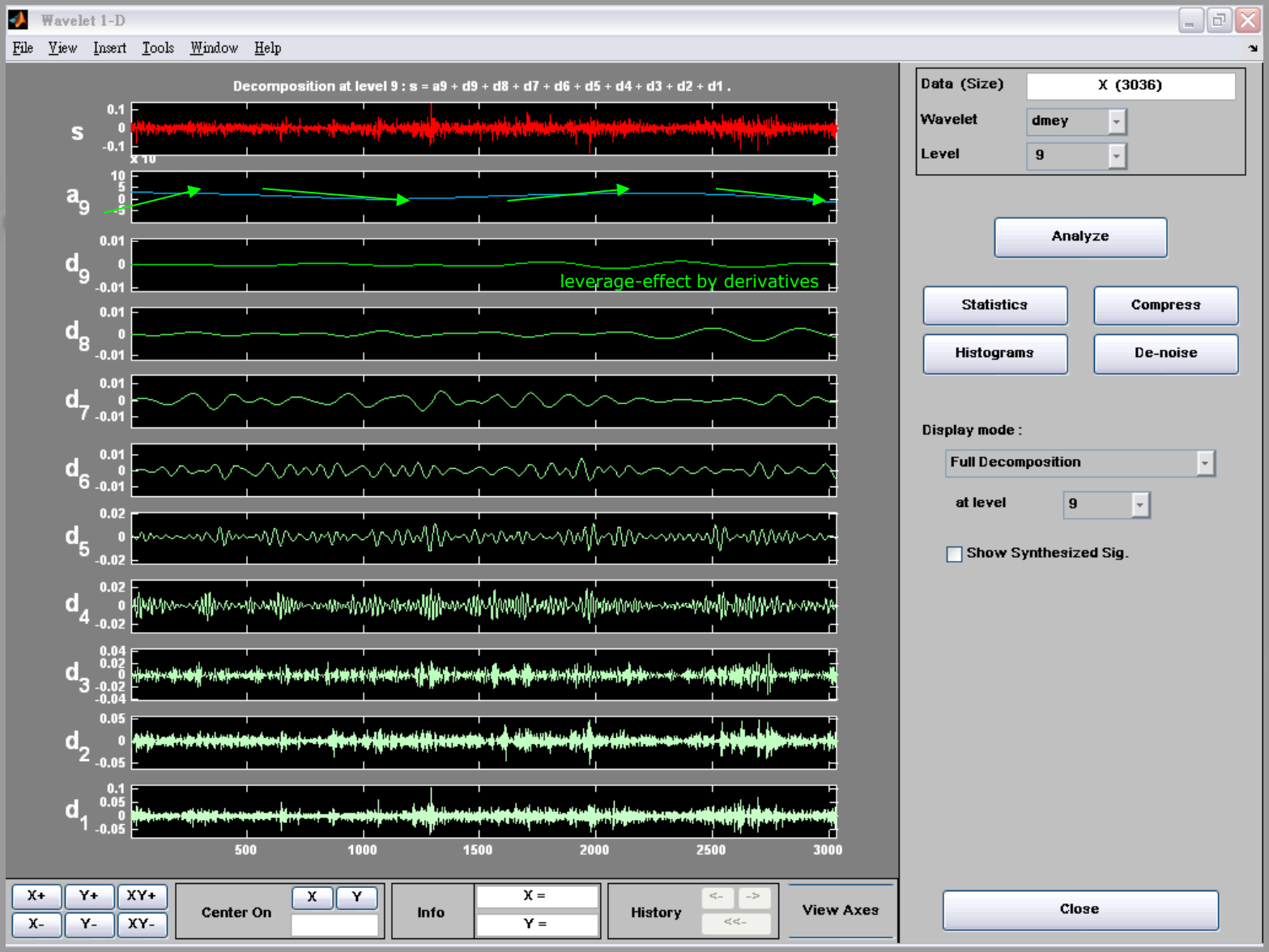

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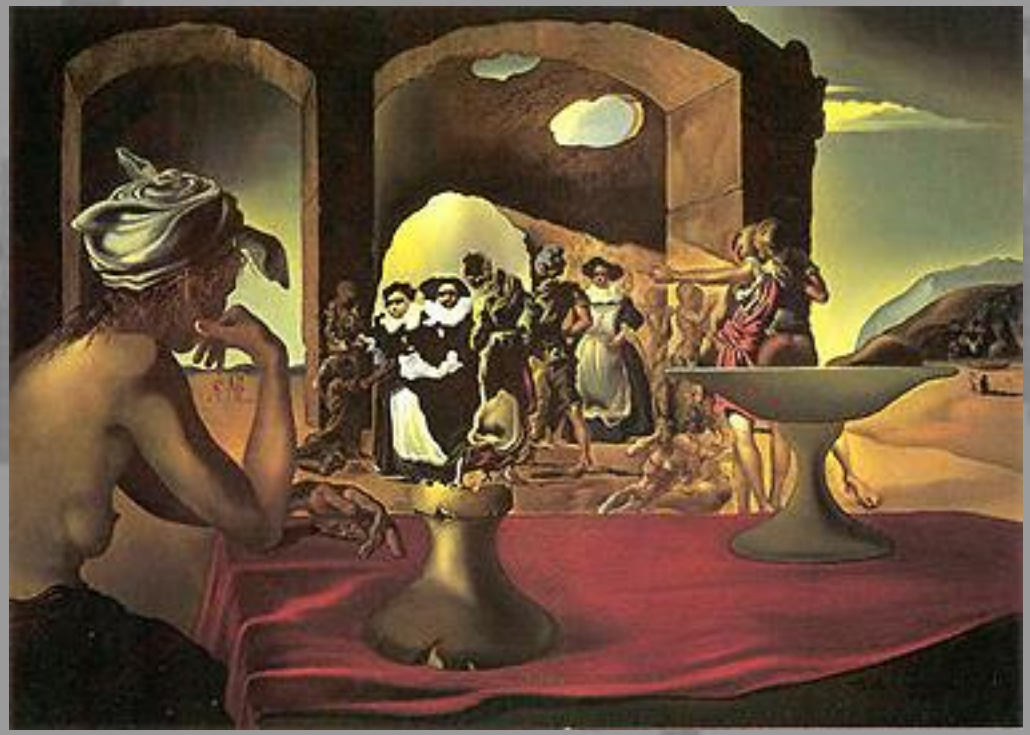

Slave Market with the Disappearing Bust of Voltaire, *Dali* (1940)

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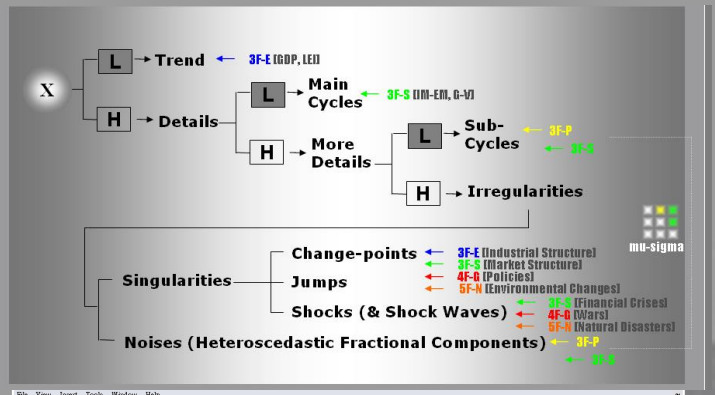
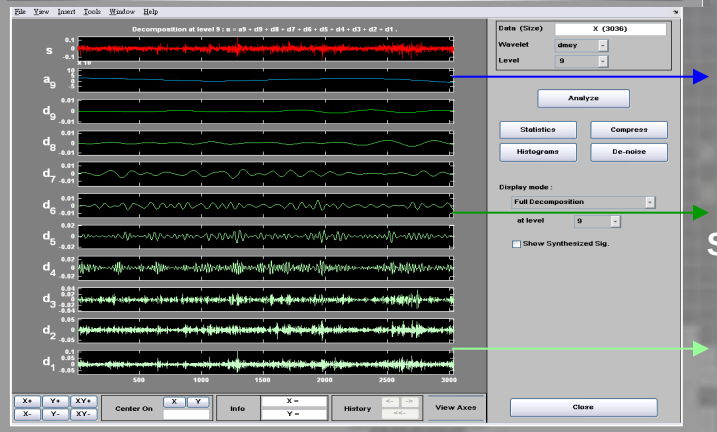
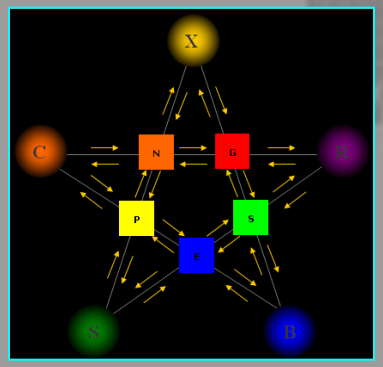






Patterns, e.g.

- Yield Inversion & Recession (GDP)
- Yield Expansion & Inflation (CPI, Commodity Price)
- Stock-Bond Correlation Variation
- Neutral Interest-Rate &* Fair Leverage-on-VaR

Structural Sense for designing Exotic* Derivatives

Dynamics & Stochastics

- External (Exogenous*) –
  - Direct Regression with Explicit Risk Factors
  - Co-integration with Implicit Risk Factors

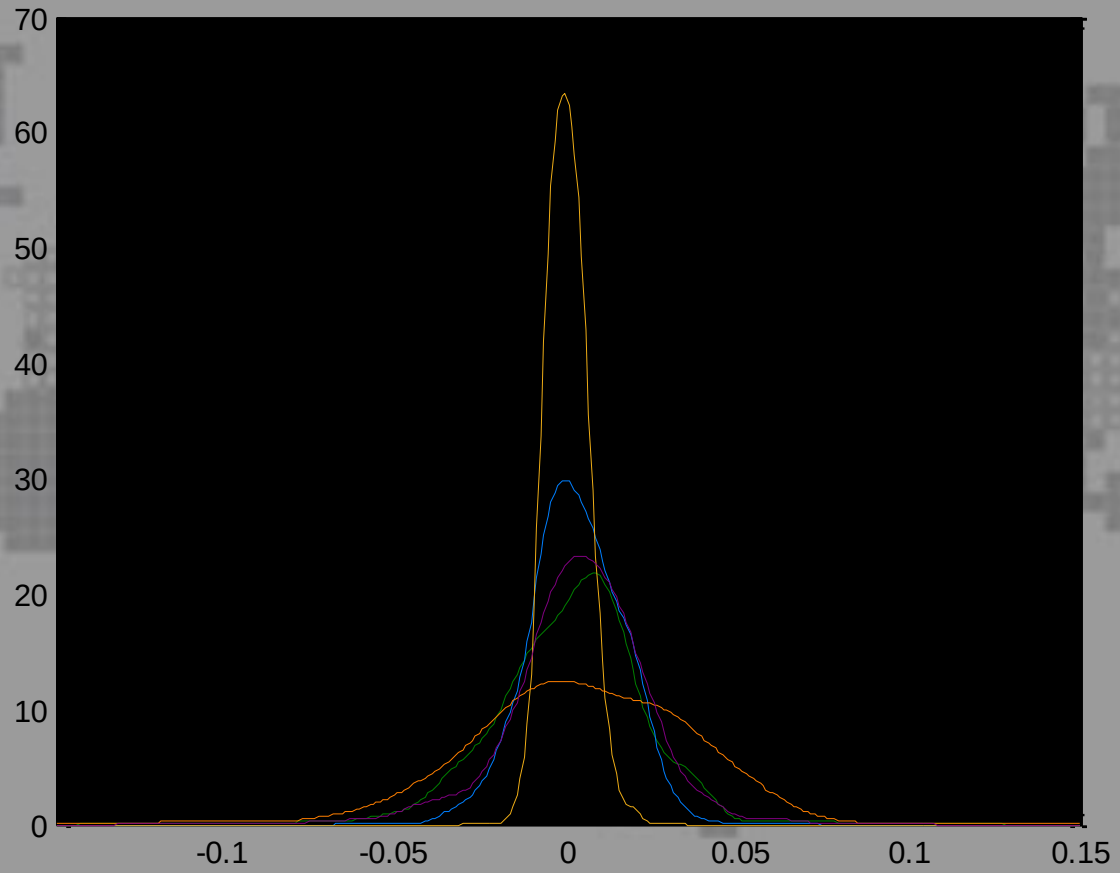
5-Risk & 5-Asset Classification → GDAA-STP

- Internal (Endogenous*) –
  - Transformation fitting Asset Nature & Product Structure
  - Auto-Regression revealing Market Pricing Mechanism

Preliminary –

- Probability Distribution Characterization
- Joint Distribution - Copula
- Correlation & Regression
- Auto-Correlation
- Co-integration - PPR

0.0004	0.0022	-0.0004	0.0034	0.0028
0.0212	0.0131	0.0058	0.0309	0.0198
-0.3696	-0.0978	0.3384	-0.4209	-0.6394
4.9570	2.9737	3.2717	3.7971	5.4063



Weekly 01/07/2000 – 03/21/2008

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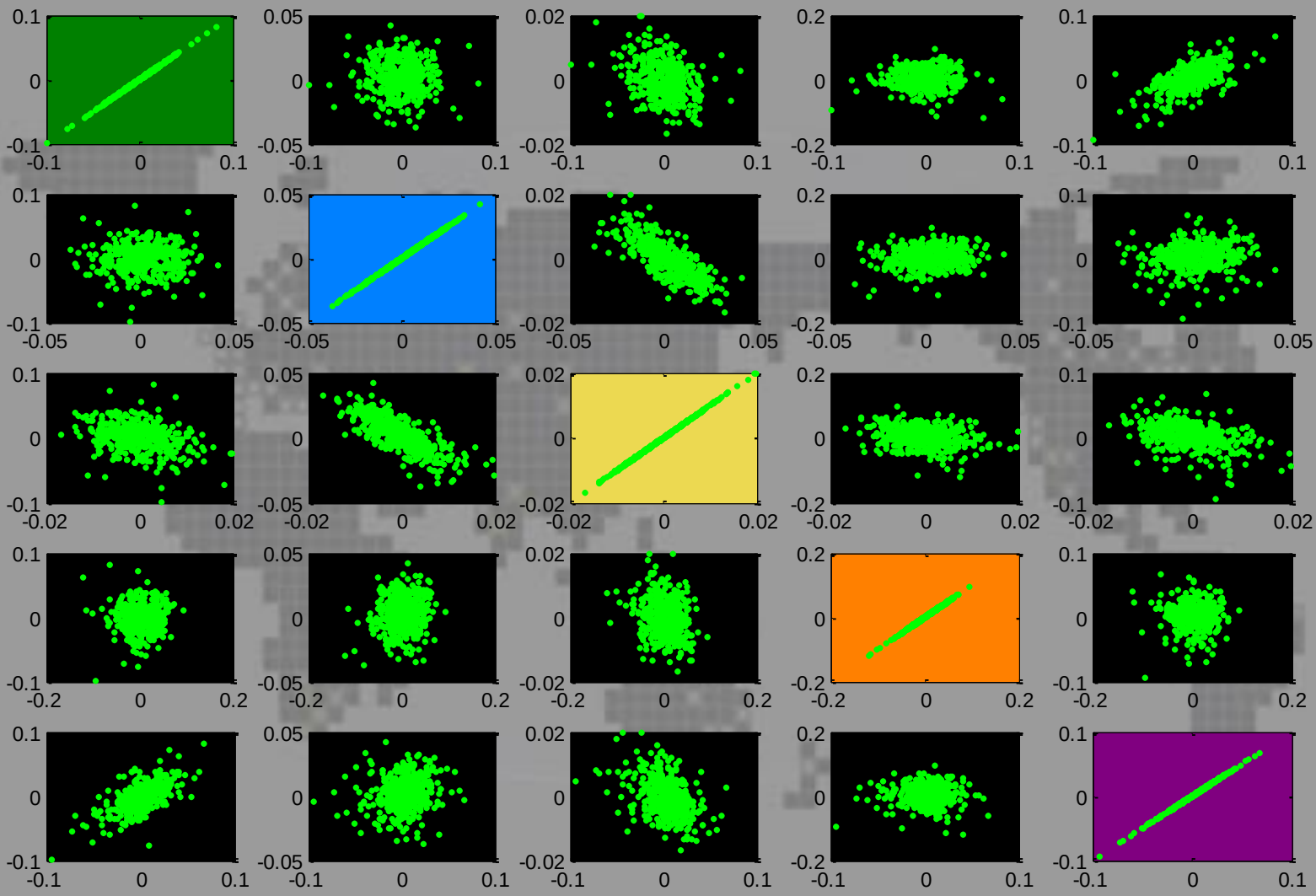

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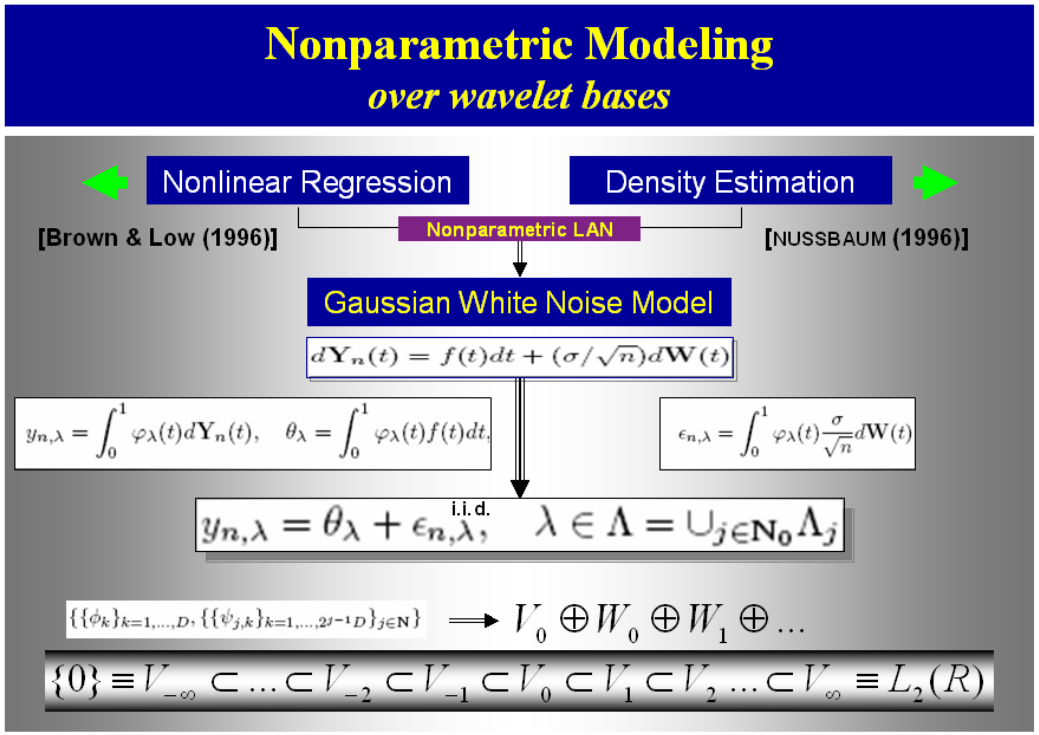
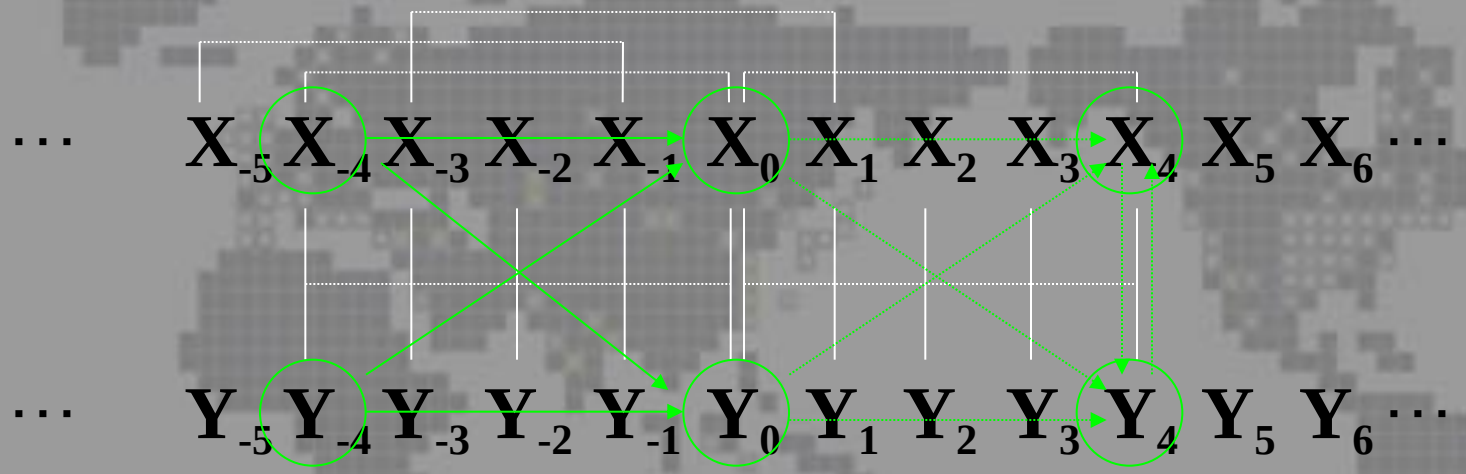
Regression = Signal Processing ?!?

from Perspectives of Abstract Inference



Financial Signal Processing based on Information Theory



Mutual Information for Prediction

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Heuristic Example for understanding Information Theory

inference via moment-generating & entropy-based information calculation

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